A reduction of the equations of the linear stationary vibrating systems without limitations of dissipation using the method of modal truncation

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1. Introduction

For computer-aided analytic research of complex vibrating systems with a large finite number of degrees of freedom, including linear stationary systems, various methods, for example, the method of finite elements and so on, are applied [1-5].

In the Paper, the mentioned linear vibrating systems are discussed upon. They are described by linear differential equations with constant coefficients. For digital integration of such equations, Runge-Kutta and other methods are applied. In a majority of cases, such systems have a very wide spectrum of natural frequencies; however, an investigator takes an interest in the much narrower range of the lowest natural frequencies within the said spectrum only. The high natural frequencies considerably increase the time of digital integration, so it is important to have a system of differential equations for describing the object under investigation where such frequencies are absent in the roots of its characteristic equations. Such equations can be obtained by reducing the number of degrees of freedom in the dynamical model of object under investigation. However, in many cases, this task is difficult or even impossible. For example, such a problem appears when the method of finite elements is applied.

Other methods of elimination of high natural frequencies are obtained on the relevant reduction of the equations describing the vibrating system [6-14]. Among those methods, the method of modal truncation where the initial system of equations is divided to a number of independent equations is widely used; each of such independent equations describes modal vibrations of one natural frequency and then the equations corresponding to high natural frequencies are eliminated (see below). The value of the error resulted by the truncation is assessed by comparing the frequency response of the vibrating system before and after the reduction in the frequency range under the interest of the investigator.

2. Initial version of the equations used for describing the system in state variables

Let’s consider that such a nonreduced system of differential equations describing vibrations of the system (object) under investigation is formed

\[
\begin{bmatrix}
[A]
\end{bmatrix}\begin{bmatrix}
\dot{q}
\end{bmatrix} + \begin{bmatrix}
[B]
\end{bmatrix}\begin{bmatrix}
\dot{q}
\end{bmatrix} + \begin{bmatrix}
[C]
\end{bmatrix}\begin{bmatrix}
q
\end{bmatrix} = \begin{bmatrix}
h(t)
\end{bmatrix}
\]

(1)

where

\[
\begin{bmatrix}
\dot{q}
\end{bmatrix} = \begin{bmatrix}
q_1, q_2, ..., q_n
\end{bmatrix}^T
\]

(2)

defining motions of the system; \(\frac{d}{dt}\begin{bmatrix}
q
\end{bmatrix}\), \(\frac{d^2}{dt^2}\begin{bmatrix}
q
\end{bmatrix}\); t is time or, more rarely, another argument; \([A],[B],[C]\) are square matrixes of the n-th degree with constant (stationary) elements; the – matrix \([A]\) may include \(r \leq n\) zero lines and columns with the same serial numbers, i.e. in the system (1), \(r\) is differential equations of the first degree that include only those generalized coordinates \(q_j\) (\(j = 1, 2, ..., r\)) and their fluxions \(\dot{q}_j\) not presented in the equation of the second fluxions \(q_j\) (Eq. (1)) can exist; \(\begin{bmatrix}
h(t)
\end{bmatrix} = \begin{bmatrix}
h_1(t), h_2(t), ..., h_r(t)
\end{bmatrix}^T\) is the vector of the n-th degree of external generalized forces involved in excitation of the system.

For examination of the vibrating system in state...
variables, the Eq. (1) are reduced to the normal (Cauchy) form, i.e. when the generalized coordinates $q_i$ are replaced for other variables, the system of equations is reduced to a system of differential equations of the first degree solved in respect of the first derivates of the said variables. Such variables are considered phase or state variables of the system described by the Eq. (1) defining all states of the system in the time $t \geq t_0$ ($t_0$ is the initial time of observance of the system). For investigation of the said equations, special softwares are used [19, 21].

The most frequently applied method of introducing variables defining the states of a system under investigation is based on replacing the derivates defining variables describing the states of a system under investigation in its state variables instead of the system of Eq. (1).

New variables are introduced

$$
x_1 = q_1, x_2 = q_2, \ldots, x_r = q_r, x_{r+1} = q_{r+1}, \ldots, x_n = q_n, \quad \text{where } n \geq r.
$$

Their vector

$$
\{x\} = \{x_1, x_2, \ldots, x_r, x_{r+1}, \ldots, x_n\}^T.
$$

To simplify the following description, it is considered that $r$ equations having no second-degree derivates of coordinates are in the beginning of the system (1), so the matrices $[A]$ and $[B]$ can be divided into the following submatrices.

$$
[A] = \begin{bmatrix}
O_{1,1} & O_{1,2} \\
O_{2,1} & A_{2,2}
\end{bmatrix}, \quad [B] = \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
$$

where $O_{1,1}$, $O_{1,2}$, $O_{2,1}$ are zero matrixes of $r \times r$, $r \times (n-r)$, $(n-r) \times r$ degree; $A_{2,2}$ is nonsingular square matrix of the $(n-r)$ degree; $[B]_1$, $[B]_2$ are matrixes of the $n \times r$ it $n \times (n-r)$ degree.

Taking into account the Eqs. (3) and (5), we obtain the following instead of the system of Eq. (1)

$$
\begin{bmatrix}
O_{1,1} & O_{1,2} \\
O_{2,1} & A_{2,2}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_r, \dot{x}_{r+1}, \ldots, \dot{x}_n
\end{bmatrix}^T +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
\begin{bmatrix}
x_1, x_2, \ldots, x_r, x_{r+1}, \ldots, x_n
\end{bmatrix}^T
+ [C] \begin{bmatrix}
x_1, x_2, \ldots, x_n
\end{bmatrix}^T = \{h(t)\}
$$

where the derivates $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_r$ are specified only formally. They are multiplied by the zero submatrixes $[O]_{1,1}$ and $[O]_{2,1}$ of the matrix $[A]$, so really they are not presented in the equations (6). These zero submatrixes are replaced for the submatrix $[B]_1$, and the derivates $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_r$ are replaced for the derivates $\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_r$. Then instead of the Eq. (6), we find

$$
\begin{bmatrix}
B_1 & O_{1,2} \\
A_{2,2}
\end{bmatrix}
\begin{bmatrix}
x_1, x_2, \ldots, x_r, x_{r+1}, \ldots, x_n
\end{bmatrix}^T +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
\begin{bmatrix}
x_{r+1}, x_{r+2}, \ldots, x_n
\end{bmatrix}^T +
[C] \begin{bmatrix}
x_1, x_2, \ldots, x_n
\end{bmatrix}^T = \{h(t)\}
$$

From it, we find

$$
\begin{bmatrix}
x_1, x_2, \ldots, x_r, x_{r+1}, x_{r+2}, \ldots, x_n
\end{bmatrix}^T =
\begin{bmatrix}
-G_1[B]_2 \begin{bmatrix}
x_{r+1}, x_{r+2}, \ldots, x_n
\end{bmatrix}^T -
-G_2[C] \begin{bmatrix}
x_1, x_2, \ldots, x_n
\end{bmatrix}^T + [G] \{h(t)\}
\end{bmatrix}
$$

here the square matrix of the $n$th degree

$$
[G] = \begin{bmatrix}
B_1 & O_{1,2} \\
A_{2,2}
\end{bmatrix}^{-1} = \begin{bmatrix}
G_1 \\
G_2
\end{bmatrix}
$$

$[G]_1$, $[G]_2$ are the submatrixes where the lines are formed of $r$ first lines and the remained $(n-r)$ lines of the matrix $[G]$. It is accepted that the inverse matrix it nonsingular and the Eq. (9) is valid. In such a case, the matrix $[A]_{2,2}$ as well as the matrix $[B]_{1,1}$ formed of $r$ first lines of the matrix $[B]_1$ should be nonsingular.

In addition, the system of Eq. (8) does not include the equations defining the values of the fluxions $\dot{x}_{r+1}, \dot{x}_{r+2}, \ldots, \dot{x}_n$. They are obtained from the part

$$
x_{r+1} = q_{r+1}, \ldots, \dot{x}_n = q_n
$$

of the Eq. (3) taking into account that $\dot{q}_{r+1} = x_{r+1}, \ldots, \dot{q}_n = x_n$. Then

$$
\begin{bmatrix}
x_{r+1} = x_{r+1}, x_{r+2} = x_{r+2}, \ldots, x_n = x_n
\end{bmatrix}
$$

After uniting the Eqs. (8) and (10), we obtain the following normal equations describing the system under examination in its state variables instead of the Eq. (1)

$$
\{x\} = [R]\{x\} + \{k(t)\}
$$

where $\{x\}$ is the vector defined by the Eq. (4); its components are the coordinates describing the location of the system in its state variables

$$
[R] =
\begin{bmatrix}
-G_1 C & -G_1 B_2 \\
O_2 & E_1 \\
-G_2 C & -G_2 B_2
\end{bmatrix}
$$
The square matrix of $2n-r=s$ degree consists of: the zero submatrix $[O]_2$ of $(n-r) \times n$ degree and the unit submatrix $[E]_1$ of $(n-r)$ degree; the submatrices $[G], [C]$ and $[G], [B]$, $r \times n$ of $r \times (n-r)$ degree; the submatrix $[G]_2, [C]$ of $(n-r) \times n$ degree and the square submatrix $[G]_2, [B]_2$ of $(n-r)$ degree

\[
[k(i)]= [G], [h(i)]
\]  

(13)

The vector of $2n-r=s$ degree; $[G]$ is the matrix of $(2n-r) \times n=s \times n$ degree obtained from the matrix $[G]$ by inserting $(n-r)$ zero lines between its submatrices $[G]_1$ and $[G]_2$.

On investigation of the vibrating system in its state variables, its output coordinates (output signals) $y_e$ ($e=1,2,...,m$) (that describe the state of the object as well) bound with its variables $\{x\}$ by linear algebraic equations are added to its equation of state $[18, 19]

\[
[y]= [D] [x] + [H] [h(i)]
\]  

(14)

where $\{y\}$ is $m$ dimensional vector ($m=1,2,...$); $[D], [H]$ are matrices of $m \times s$ and $m \times n$ degree with constant or varying in course of time elements; the values and character of variation of such elements depend on a system under investigation.

3. The proposed version of the equations of state variables and their reduction

The obtained mathematical model Eqs. (11) and (14) of the system under investigation in its state variables is based on the application of variables for defining the state of the system provided in the Eq. (4).

As it were mentioned above, on investigation of the vibrating systems described herein (Eq. (1)), it is not reads Eq. (11) because of a considerable time of integration, when the spectrum of natural frequencies of the system includes the range of high natural frequencies (out of the interests of the investigator), not only the range of low natural frequencies. It would be purposeful to reduce the Eq. (11) by eliminating components with high natural frequencies from their solutions and simultaneously maintaining an adequacy of the obtained results with a permissible error.

For such reduction of the Eq. (11) and saving the computer time, it is proposed in this paper to use the normal Bulgakov’s coordinates for variables of the system’s state of variables and the method of modal truncation on the base of [20, 21]. For this purpose, the below-described procedure is used.

The method is applicable, if the matrix $[R]$ has multiple natural values; in such a case, the element divisors related to them should be linear (this condition is equivalent to the condition on absence of any secular terms in the solutions of homogenous Eqs. (1) and (11), when $\{h\}=0$.

In addition, the vector $\{x\}$ should have no zero components, i.e. the systems of Eqs. (1) and (11) should include no algebraic equations. It is considered that the said conditions were satisfied (if secular members appear in solutions of the equations, they usually may be avoided by a slight correction of the dynamic model of the system under examination without losing its adequacy).

It is considered that the matrix included in the Eq. (11) has $s$ natural values $\lambda_j$ ($j=1,2,...,s$), including $s''$ real natural values $\lambda_{\sigma}=\lambda_{\sigma}$ ($\sigma=1,2,...,s''$) and $s'$ couples of complex conjugates roots $\lambda_{\sigma+i\omega} = \omega_i + i\omega_b$, $\lambda_{\sigma+i\omega} = \omega_i - i\omega_b$; where $\omega_i$, $\omega_b$ are real and imaginary parts of roots, $h=1,2,...,s$.

The natural values $\lambda_j$ of the matrix $[R]$ are also the roots of the characteristic equations of the systems of Eqs. (1) and (11), the values $\omega_t$ is the natural frequencies of the system under investigation; $\epsilon_i$ defines damping of free vibrations with frequency $\omega_t$ and $\lambda_\sigma$ defines apriorical nonvibrating processes. It can be seen that $s=s'+2s''=2n-r$.

Each real natural value $\lambda_{\sigma}$ of the matrix $[R]$ corresponds to its own vector $\{V\}_\sigma = \{\nu\}_\sigma$ with $s$ components, each complex couple of roots $\lambda_{\sigma+i\omega}$ and $\lambda_{\sigma+i\omega}$ to own vectors $\{V\}_{\sigma+i\omega} = \{\nu\}_{\sigma+i\omega} + i\{\nu\}_{\sigma+i\omega}$ with $s$ components and, correspondingly, $\{V\}_{\sigma+i\omega} = \{\nu\}_{\sigma+i\omega} - i\{\nu\}_{\sigma+i\omega}$. The natural values $\lambda_{\sigma+i\omega}$ and the own vectors $\{V\}_{\sigma+i\omega}$ corresponding to them hereinafter will not be used.

Each natural vector $\{V\}_{\sigma+i\omega}$ is simultaneously the mode of natural vibrations corresponding to the natural frequency $\omega_t$ of the homogeneous part of the system under investigation Eq. (1) according to the generalized coordinates $q_1,q_2,...,q_a$ and the mode of speeds of natural vibrations according to the derivates of the generalized coordinates $q_{\sigma+1},q_{\sigma+2},...,q_a$. The complex form of these vectors shows that they were formed taking into account the damping in the system; however, no special requirements are set for the character of the latter [20, 21]. Each natural vector $\{V\}_\sigma$ is simultaneously the mode of apriorical motions corresponding to the real root $\lambda_{\sigma}$ of the homogeneous part of the system under investigation (1) according to the generalized coordinates $q_1,q_2,...,q_a$ and their derivates $q_{\sigma+1},q_{\sigma+2},...,q_a$.

When the natural vectors are known, a square modal matrix of the $s$-th degree $[\nu]$ consisting of $s'$ columns of the vectors $\{V\}_\sigma$ and $2s''$ columns of the real $\{V\}_{\sigma+i\omega}$ and imaginary $\{V\}_{\sigma+i\omega}$ parts of the vectors $\{V\}_{\sigma+i\omega}$ is formed [15]. When the said columns are laid out in the way where $s'$ first columns of the matrix $[\nu]$
are the vectors \(\{v\}_\sigma\) and then couples of the vectors \(\{v\}_{\sigma,h}\) and \(\{v\}_{\sigma,h}^*\) follow, the following structure of
\[
\begin{bmatrix}
\{v\}
\end{bmatrix}
\] (15)

is the vector of \(s\)th degree in the normal Bulgakov's coordinates (NBC).

After inserting the value Eq. (16) of the vector \(\{x\}\) into the Eq. (11), we find [15]
\[
\begin{bmatrix}
\{\xi\}
\end{bmatrix} = \{\mathcal{Q}\}\{\xi\} + [z] [h(t)]
\] (18)

where, if the order of components of the vector \(\{\xi\}\) provided in the Eq. (17) is preserved
\[
\{\mathcal{Q}\} = [v]^T [R] [v] = \begin{bmatrix}
\{X\} & O_1 \\
O_4 & \{\Omega\}
\end{bmatrix}
\] (19)

\(\{X\}, \{\Omega\}\) are zero submatrices of the \(s' \times 2s'\) and \(2s' \times s'\) degree;
\[
\{z\} = \text{diag} \{\chi_1, \ldots, \chi_s\}
\] (20)

\(\{\Omega\}\) is the block square submatrix of the \(2s'\) degree; square submatrices
\[
\begin{bmatrix}
\varepsilon_h & \omega_h \\
-\omega_h & \varepsilon_h \\
\end{bmatrix}, h = 1, 2, \ldots, \sigma
\] (21)

are situated along its diagonal and all remained elements of the said submatrix are zero elements;
\[
[z] = [v]^T [G].
\] (22)

matrix of the \((2n-r) \times n = s \times n\) degree.

The scalar expression of the Eq. (18) will be as follows. To each real root \(\chi_\sigma\), an equation not bound with other equations
\[
\xi_\sigma - \chi_\sigma \zeta_\sigma = \Phi_\sigma(t), (\sigma = 1, 2, \ldots, \lambda)
\] (23)

will correspond, and to each complex joint root \(\varepsilon_h \pm i \omega_h\), a couple of equations not bound with other equations
\[
\begin{bmatrix}
\varepsilon_h - \omega_h \\
\omega_h & -\varepsilon_h
\end{bmatrix} \Phi_\sigma(t); \Phi_\sigma(t); (h = 1, 2, \ldots, \varepsilon
\] (24)

will correspond, where
\[
\Phi_\sigma = \sum_{h=1}^{\varepsilon} z_{\sigma,h} h \quad \Phi_{\sigma,h} = \sum_{h=1}^{\varepsilon} z_{\sigma,\sigma,h} h
\] (25)

So, the initial system of equations was divided to independent equations with easily findable solutions. The variables \(\{x\}\) as well as the coordinates \(\{q\}\), if the normal Bulgakov’s coordinates \(\{\xi\}\) are known, should be found from the Eq. (16). It should be noted that the division of the equations of the system under investigation into independent equations was carried out without applying any limitations to the structure and elements of the matrix \([B]\) included in an equation of the system (1).

The other version of equations of the system under investigation in the state variables consists of the system of Eq. (18) with the joined system of algebraic linear Eq. (14) where the value of the vector \(\{x\}\) is taken from the Eq. (16)
\[
[\{y\}] = [D] [v] [\{\xi\}] + [H] [h(t)]
\] (26)

This version differs from the above-mentioned one (see the Eqs. (11) and (14)) in a considerably simpler structure of the differential equations; in addition, it provides an opportunity of a major reduction of these equations using the method of modal truncation.

It may be made sure that upon investigating of a dynamical system in its state variables and solving the equations that describe it by numerical integration, the process of the integration will be longer when the natural frequencies of the system are higher and processes of higher frequencies are explored (the step of integration becomes smaller). If the range of natural frequencies of the system under investigation is very large and the investigator takes an interest in the range of low frequencies only, the process of integration can be shortened considerably by eliminating the systems of Eqs. (24) that correspond to
higher natural frequencies \(\omega_h\) from the process of calculation, i.e. by applying the method of modal truncation. In addition, often all Eq. (23) or a part of them can be neglected. In such a case, instead on the matrix (19) and the system of Eq. (18), we obtain

\[
\begin{bmatrix}
Q_0 & O_s \\
O_s & \Omega_s
\end{bmatrix}
\]

(27)

\[
\begin{bmatrix}
\xi_* \\
\hat{\xi}_*
\end{bmatrix} = \begin{bmatrix} Q_0 \end{bmatrix} \begin{bmatrix}
\xi \\
\hat{\xi}
\end{bmatrix} + \begin{bmatrix} z \end{bmatrix} \{h(t)\}
\]

(28)

where \(\{\xi\}\) is the vector obtained from the vector \(\{\xi\}\) after elimination of the neglected NBCs; \(\chi\), \(\Omega\), \(Q\) are square submatrices of the \(\alpha\) and \(2\beta\) degree obtained from the submatrices (19) and (20) when a part of the Eqs. (23) and a part of the couples of Eqs. (24) are neglected; \(\alpha < s', \beta < s''\) – the number of the remained Eqs. (23) and, correspondingly, the remained couples of Eqs. (24); \(\begin{bmatrix} O_0 \end{bmatrix}, \begin{bmatrix} O_s \end{bmatrix}\) are zero submatrices; \(\begin{bmatrix} Q_0 \end{bmatrix}\) is the square matrix of \((\alpha + 2\beta)\) degree analogous to the matrix \(\begin{bmatrix} Q_0 \end{bmatrix}\), but without the excluded elements of the Eqs. (23) and (24); \(\begin{bmatrix} z \end{bmatrix}\) is the matrix of \((\alpha + 2\beta)\times n\) degree obtained from the matrix \(\begin{bmatrix} z \end{bmatrix}\) after elimination of the lines with the serial numbers corresponding to the eliminated coordinates \(\xi_{s'}'\), \(\xi_{s''+1}'\), \(\xi_{s'''+1}'\).

\[\xi = \begin{bmatrix}
\chi \\
\Omega
\end{bmatrix}
\]

\[\hat{\xi} = \begin{bmatrix}
\hat{\chi} \\
\hat{\Omega}
\end{bmatrix}
\]

The approximate values of the variables \(x_p\), i.e. \(x_p'(p = 1, 2, ..., s)\), are found from the equation that is analogous to the Eq. (16)

\[
\begin{bmatrix}
x'_p \\
\hat{x}'_p
\end{bmatrix} = \begin{bmatrix} \nu \end{bmatrix} \begin{bmatrix} \xi \end{bmatrix} + \begin{bmatrix} h(t) \end{bmatrix}
\]

(29)

where \(\{x'_p\}\) is the vector of the approximate variables \(x'_p\) of the state variables, \(\{\nu\}\) is the reduced modal matrix of \(s\times(\alpha + 2\beta)\) degree found from the matrix \(\begin{bmatrix} \nu \end{bmatrix}\) where only the columns corresponding to the assessed natural values \(\lambda_j\) of the matrix \(\begin{bmatrix} R \end{bmatrix}\) are left.

The \(v\) - dimensional vector \(\{y'\}\) of approximate coordinates of the output of the system is found from the equation that is analogous to the Eq. (14)

\[
\begin{bmatrix}
y' \\
\hat{y}'
\end{bmatrix} = \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} v' \end{bmatrix} + \begin{bmatrix} H \end{bmatrix} \{h(t)\}
\]

(30)

After insertion of the value of the vector \(\{x'_p\}\) from Eq. (29) into the equation, we find

\[
\begin{bmatrix}
y' \\
\hat{y}'
\end{bmatrix} = \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} v \end{bmatrix} \begin{bmatrix} \xi \end{bmatrix} + \begin{bmatrix} H \end{bmatrix} \{h(t)\}
\]

(31)

So, the reduced version of the equations of the system under investigation with NBCs in the state variables consists of the systems of Eqs. (28) and (31).

The issue of the numbers \(\alpha\) and \(2\beta\) remained in NBC \(\xi_j\) is important. In a majority of cases, a reduction of the number \(\alpha\) of the Eqs. (23) can be avoided and only
Fig. 2 The scheme of the dynamic model of the example

The number \( \beta \) of the Eqs. (24) can be reduced; however, a specific decision should be passed for each individual system under investigation. For defining the number of the remained NBC, it is proposed to compare the amplitude-frequency responses (AFRs) and the phase responses (PRs) obtained upon certain selection of the components of the vector \( \{ h(t) \} \) for the non-reduced system (1) or (11), (18) with the respective responses of the reduced system (28) calculated in the range of frequencies the investigator is interested in. Nonzero components of the vector \( \{ h(t) \} \) can be following

\[
h_a(t) = A_a \sin vt + B_a \cos vt
\]

(\( a = 1, 2, \ldots, n \))

here the values of the constant the coefficients \( A_a \), \( B_a \) and the range \( \Delta \nu \) of changing of the excitation frequencies \( \nu \) are chosen by the investigator. Upon comparison of AFRs and PRs of the whole and reduced systems, the value of inadequacy is a base for making a conclusion on the level of the errors of the reduction and the required number of remained NBC. One of the possible algorithms for AFRs and PRs calculation that does not require much time for the calculation is provided in [15]. The algorithm is based on the application of solutions of the equations (18) in the analytic form and the equation (16) in the case of harmonic excitation of the system. The scheme of formation of the reduced equations in Bulgakov’s coordinates is shown in Fig. 1.

4. Example

In our paper [22], a system of equations of the type Eq. (18) for investigating transversal vibrations of the unit of plate cylinders and blanket cylinders in a section of offset printing press was described and used. Using the results of the said work, we’ll show an efficiency of the proposed reduction. The above-mentioned unit (Fig. 2) consists of two plate cylinders 1, 2 with printing forms attached to them (Fig. 2, not shown herein) and two blanket cylinders 3, 4 covered by a special elastic textile (blanket) 5. The blankets are fixed to the said cylinders by special oblong locks along the generatrices of the cylinders.

Fig. 3 Transverse vibrations of the middle of the blanket cylinder of the model: a, b – the amplitude and phase responses versus frequency (1 – for the whole system, 2 – for the simplified system); c, d – vibrations caused by shocks of the blanket fixing locks for the whole system (c) and the simplified system (d)
The length of all cylinders is 1040 mm, the diameter - 200 mm. The printing process runs as follows: while the cylinders connected with gearwheels 7 and pressed against each other along their generatrices rotate and deform the blanket, the prints from the ink-moistened printing forms are transferred to the blankets and from them – to the both sides of the paper tape 8 moving between the blanket cylinders 3, 4. The position of the blanket cylinders 3, 4 pressed against each other through the blanket 5 is regulated in such a way that ensures getting of the locks 6 into collision while rotation of the cylinders. It causes blows and transversal vibrations of the cylinders pressed against each other (similar vibrations appear on getting of the locks into collision with surfaces of plate cylinders; however, the intensity of such vibrations is lower). For description of the vibrations, the equations of the type (1), then also the equations of the type (11), (18) are used; in [17], these equations are further used nonreduced. For formation of the equations, all cylinders are divided into finite elements. In addition, elasticity and damping of the blankets are taken into account. Thus, a system with 168 degrees of freedom is formed. Its natural frequencies are situated in the 171.5 - 530 Hz range. Blows are simulated by rectangular 100 N force impulses with duration of 0.003 s (the period 0.1 s); the said impulses impact the pressed against each other plate cylinders in the radial direction in units of finite elements entering into contact in the zones of the plate cylinders (each of two plate cylinders is impacted by 10 impulses). The transversal vibrations generated by the said impulses in the middle of the plate cylinder 2 are calculated.

It is accepted that a person engaged in the calculation takes an interest in vibrations with the frequencies up to 700 Hz, so only 12 couples of equations in normal Bulgakov’s coordinates are left in the reduced system.

They correspond to all natural frequencies of the system up to 750 Hz.

Amplitude-frequency and phase responses of the complete and the reduced system are calculated for the range 0-1000 Hz, where the force impulses are replaced by harmonic excitation. The calculations were carried out using the mathematical simulation set MATLAB. When the equation (11) is applied for the calculations, the time of numerical integration equals to 14 min 14 s (the step 5e-7), and when the reduced equation of the type (30) is applied, it equals to 8 s (the step 1e-5), i.e. the time reduces 106 times. The conversion of the system (11) into equations of the type (18) took 1.5 s, and the calculation of the frequency responses took 2 s.

The obtained results are shown in the Fig. 3. It can be seen that the error of the reduction in the range 0-750 Hz is very small.

5. Conclusions

1. For shortening the time of numerical integration of the differential equations usable for describing linear stationary vibrating systems with a wide spectrum of their natural frequencies, it is purposeful to convert the said equations in to the normal Bulgakov’s coordinates and then to reduce them eliminating the equations to high natural frequencies by the method of modal truncation.

2. For the proposed method of calculation, no limitations for dissipation of the vibrating system are required.

3. The value of the error appeared during the reduction is assessed by a comparison of the amplitude-frequency and phase responses of the system under investigation obtained from the initial and reduced equations with harmonic excitation.

References


8. Documentation for ANSYS. Analysis Tools (Damping Matrices; Mode Superposition Method; Reduced Order Modeling of Coupled Domains).


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A REDUCTION OF THE EQUATIONS OF THE LINEAR STATIONARY VIBRATING SYSTEMS WITHOUT LIMITATIONS OF DISSIPATION USING THE METHOD OF MODAL TRUNCATION

Summary

A method for reduction of differential equations describing linear stationary vibrating systems with a finite number of degrees of freedom usable for shortening the time of digital integration of such equations, when the dynamic model of the vibrating system has a wide spectrum of natural frequencies and the investigator takes an interest in the range of low natural frequencies within the said spectrum only, is proposed herein. For this purpose, the equations are formed in the state variables of the system using the normal Bulgakov’s coordinates and then are reduced by rejecting the equations bound with higher natural frequencies according to the method of modal truncation. The method does not require any limitations of damping of the systems under the investigation. In the example, it is shown that in case of a system of 168 degrees of freedom, the time of digital integration was reduced after simplification about 100 times upon small reduction errors.

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