A new algorithm for cylindrical worm gears dimensioning based on the hydrodynamic lubrication conditions between the teeth flanks

Antal Tiberiu Alexandru
The Technical University from Cluj-Napoca, C. Daicoviciu no. 15, 400020, Cluj-Napoca, Romania, E-mail: Tiberiu.Alexandru.Antal@mep.utcluj.ro

1. Introduction

A method for increasing the lifetime of the cylindrical worm gears is to ensure conditions of hydrodynamic lubrication between the flanks of the meshing teeth. In the paper the main parameters of the cylindrical worm gears are determined taking into account the load conditions and some parameters connected to the lubrication.

2. Cylindrical worm gear module calculation based on hydrodynamic lubrication

For cylindrical worm gears with hydrodynamic lubrication used in the mechanical transmission field, based on the expressions from [1, 2] the module can be determined by the following relation

\[
m_x \geq \frac{2}{q + z_2 + 2x} \sqrt[1.15]{\frac{T_2}{21\eta 0.6 C_{\alpha} 0.7^2 n_1 0.5^2 E_{red}}} \quad (1)
\]

where the lubrication thickens \( h^* = 0.018 + \frac{q}{7.86(q + z_2)} \) + \( \frac{z_2}{110} \cdot \frac{36300}{370.4} + \frac{2(0.5 + q + 1)}{213.9} \cdot q \) the diameter factor, \( z_1 \) the number of the threads (starts) of the worm, \( z_2 \) the number of the teeth of the worm wheel, \( x \) the specific addendum modification of the wheel, \( T_2 \) the moment of torsion on the axle of the worm wheel in Nm, \( \lambda \) the safety coefficient (\( \lambda = 1, \ldots, 2 \)), \( R_{12} \) and \( R_{12} \) the arithmetic average roughness on the teeth of the worm and on the worm wheel measured in \( \mu \)m (\( R_{12} = 0.4 \mu \)m at corrected worm and \( R_{12} = 1.6 \mu \)m for the milled worm wheel), \( C_{\alpha} = 1.7 \times 10^{-6} \) m²/N is the viscosity pressure variation exponent in the case of mineral oils, \( \eta_{OM} \) the oil viscosity at ambient pressure and at temperature of the meshing area in Ns/m², \( n_1 \) the angular speed of the worm in min⁻¹, \( E_{red} = 140144 \) MPa is the elastic modulus of the teeth (for bronze worm wheel CuSn12 and steel worm).

The \( m_x \) module and the \( q \) diameter factor determined from Eq. (1) must satisfy the strength conditions of the transmission and the efficiency should be as high as possible.

3. Lubricated worm gears efficiency

If the lubrication conditions are accomplished between the teeth flanks during the meshing the efficiency can be calculated with the formula from [3] as

\[
\eta = \frac{1}{1 + \frac{\mu V_{12}}{cos(\alpha_n) V_1 cos(\beta_1)}} \quad (2)
\]

where \( \mu \) is the friction coefficient between the flanks of the teeth in contact, \( \alpha_n \) the worm tooth profile angle at normal section (\( \alpha_n \) can be considered 20°), \( V_{12} \) relative velocity between the teeth flanks on the rolling cylinder of the worm, \( V_1 \) peripheral speed of the worm on the rolling cylinder of the worm, \( \beta_1 \) worm tooth declination angle on the rolling cylinder.

For worm gears with the wheel made of bronze CuSn12 and the worm from hardened steel HRC > 45. The friction coefficient can be calculated using the relation in [3]

\[
\mu = \frac{0.04}{V_{12}^{0.50}} = \frac{0.04}{\frac{\pi m q n_1}{60 \times 1000} \sqrt{z_1^2 + q^2}} \quad (3)
\]

Fig. 1 Teeth contact between the worm and the worm wheel in the normal section

Considering the contact point C (Fig. 1) where the common perpendicular between the axes stings the division cylinder of the worm, using relations (2) and (3), the following relation can be established for the efficiency

\[
\eta = \frac{z_2 q \cos(\alpha_n)}{z_2 q \cos(\alpha_n) + \frac{0.04(z_1^2 + q^2)}{\sqrt{\frac{\pi m q n_1}{60 \times 1000} \sqrt{z_1^2 + q^2}}} \quad (4)
\]
4. Contact pressure check for the module determined by the hydrodynamic lubrication conditions

The contact pressure between the teeth flanks can be determined by the Hertz formula given in [4, 5]

\[ \sigma_H = \frac{F_n \rho_1 + \rho_2}{L_k \rho_1 \rho_2 \pi \left( \frac{1}{E_1} + \frac{1}{E_2} \right)} \leq \sigma_{Hw} \]  

(5)

where \( F_n \) is the normal force on the tooth flank in the normal section of the worm tooth (Fig. 1), \( L_k \) the length of the contact line between the teeth flanks in contact \( (L_k = 0.55mq) \), \( \rho_1 \) and \( \rho_2 \) the radii of curvature of the profiles in contact in normal section, \( \nu_1 \) and \( \nu_2 \) the Poisson coefficients for the material of the worm and of the wheel \( (\nu_1 = 0.30 \text{ for the steel worm and } \nu_2 = 0.35 \text{ for the bronze worm, } E_1 \text{ and } E_2 \text{ the elastic modules of the worm and the wheel } (E_1 = 2.1 \times 10^5 \text{ MPa for steel worm and } E_2 = 0.883 \times 10^5 \text{ MPa for the bronze CuSn12 wheel)}, \sigma_{Hw} \) the admissible contact pressure of the wheel from the worm gear \( (\sigma_{Hw} = 425 \text{ MPa}) \).

![Fig. 2: Forces acting on the worm and the worm wheel in the contact point](image)

5. Worm shaft bending check

As shown in [6, 7] bending of the worm shaft can be determined using

\[ f = \frac{l^4}{48E_im_1} \sqrt{F_{r1}^2 + F_{t1}^2} \leq f_a \]  

(9)

where \( l \) is the distance between the supports of the worm \( (l = \psi \alpha, \text{ generally } \psi_a = 1.5, \ldots, 2) \), \( a = m_s(q + 2z_2 + 2x)/2 \) the distance between the axes of the worm and wheel in [mm], \( I_m = \pi d_m^4/64 \) geometric moment of inertia in mm\(^4\), \( d_m \) reference diameter of the worm shaft, \( F_{r1} \) radial force on the denture of the worm, \( F_{t1} \) tangential force on the denture of the worm, \( f_a \) admissible bending deformation.

From Fig. 2 the \( F_{r1} \) and \( F_{t1} \) forces can be computed using the following relations

\[ F_{r1} = \frac{2T_1}{d_m} \]  

\[ F_{t1} = \frac{2T_1}{m_s q} \]  

(10)
where \( T_1 \) is the torque on the worm shaft.

Taking into account the presented relations the bending of the worm can be computed from Eq. (9) thus

\[
f = \frac{y_2^2}{3\pi\mu I_{n2}} \left( q + z_2 + 2x \right) \frac{z_1}{m^2_q} \frac{T_2}{z_2} \eta
\]

where \( T_2 = \eta T x_2 / z_1 \) is the torque on the worm wheel, \( f_o \) the admissible bending deformation \([4]\) \( (f_o = 0.004 m_o \) at hardened worm and \( f_{mx} = 0.01 m_o \) at improved worm).

6. Algorithm implementation and numerical results

Matlab was used to find the main parameters of the cylindrical worm gears that satisfy the required hydrodynamic lubrication conditions, bearing capacity and stiffness at curving using the following algorithm:

\[
k = 1;
A = [];
\]

for \( q = 7:1:17 \)

for \( x = -1:0.1:1 \)

\( \eta = \text{relation \( (4) \)} \)

\( f = \text{relation \( (13) \)} \)

\( mx = \text{relation \( (1) \)} \)

\( fm = \text{right part of relation \( (8) \)} \)

\% \( f_c = 0.01 \times mx \) - improved worm

\% \( f_c = 0.004 \times mx \) - hardened worm

\( \text{if} \ (\eta > \text{eta_min}) \& \& \ (f <= 0.01 \times mx) \& \& \ (mx >= fm) \)

\( A = [A; \{q \times \eta \times f \times 0.01 \times mx \times fm \}] \);

\( k = k + 1 \);

end;

end;

try

\( \text{B = sortrows}(A, 3) \);

for \( i = 1:k \)

\( \text{fprintf} \left ( \text{‘%f \ ... \ %f \n’}, B(i,1), \ldots, B(i,7) \right ) \);

end;

catch exception

\( \text{disp(exception.message)} \);

end;

The results from Table 1 and Table 2 were obtained for the following input data: \( z_1 = 1, z_2 = 41 \), eta_min = 0.85, \( \eta_{out} = 0.08 \text{ Ns/m}^2 \), \( m_o = 1000 \text{ mm}^{-1} \), \( T_o = 587 \text{ Nm} \), \( \lambda = 1 \), \( y_2 = 1.5 \). Two for cycles are used to control the variation of \( q \) and \( x \) main parameters. The \( q \) is changing between 7 and 17 with step 1 and \( x \) between -1 and 1 with step 0.1. The values of \( m_o \) are computed using the \( (1) \) expression and stored in the \( mx \) variable. Only the results that have the efficiency \( \text{eta \ variable \) computed using Eq. \( (4) \) higher than the given eta_min value and that are satisfying Eqs. \( (8) \) and \( (13) \) are stored in the A matrix. These rows are sorted ascending by the efficiency column (the third column in the A matrix), stored in B matrix and then printed. The bending from Eq. \( (13) \) is given in the \( f \) (left part of the inequality) and \( f \) (right part of the inequality) columns, while the module from Eq. \( (8) \) is given in the \( fm \) (is the right part of inequality). Exceptions must be handled using try … catch as the restrictions can leave the A matrix without any rows, in which case the sorting would cause error.

7. Conclusions

Based on the numerical results from Table 1 and Table 2 in the case of the cylindrical worm gears of ZI, ZA, ZN and ZK type, with hydrodynamic lubrication between the teeth flanks, the axial module \( \lambda \) is chosen from the \([-1, -0.2]\) domain. From Table 2, where the worm is improved, the values for the specific addendum modification can be chosen from the \([-1, -0.2]\) domain. Numerical results confirm the hypothesis of the literature \([5]\) as to have hydrodynamic lubrication between the flanks of the teeth, at worm gears, the specific addendum modification is good to be negative. With the help of these values the geometrical dimensions of the gear can be determined while also having knowledge of the efficiency.

### Table 1

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<tr>
<th>( q )</th>
<th>( x )</th>
<th>( \eta )</th>
<th>( f_c )</th>
<th>( f_o )</th>
<th>( m_o \times \lambda )</th>
<th>( fm )</th>
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References


Antal Tiberiu Alexandru

NAUJAS CILINDRINĖS SLIEKINĖS PAVAROS MATMENŲ NUSTATYMO ALGORITMAS, ĮVERTINANTIS HIDRODINAMINDELFILTĖS KRUMPLIŲ ŠONŲ TEPIMO SĄLYGAS

R e z i u m ė

Straipsnyje aprašytas naujas būdas cilindrinės sliekinės pavaro geometriniai parametrams nustatyti, įvertinant tepalo plevės susidarymą ant krumplių šonų. Atsižvelgiant į tai, nustatytas ašinis modulis mx ir kiti parameitai, užtikrinantys gerą sliekinės pavaro efektyvumą bei patenkinantys grūdių apkrovimo ir slieko lenkimo sąlygas. Pagrindinių parametrų skaitiniams rezultatams nustatyti naudota programa Matlab.

Antal Tiberiu Alexandru

A NEW ALGORITHM FOR CYLINDRICAL WORM GEAR DIMENSIONING BASED ON THE HYDRODYNAMIC LUBRICATION CONDITIONS BETWEEN THE TEETH FLANKS

S u m m a r y

The paper gives a new method for obtaining the geometrical dimensions for cylindrical worm gears based on the conditions of forming a lubrication film between the teeth flanks. Based on this requirement the axial module mx is determined together with other parameters so that the worm gear will have good efficiency while the bearing strength and worm shaft bending conditions are satisfied. Numerical results were obtained for the main parameters using the Matlab programming environment.

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