Strength and fracture criteria application in stress concentrators areas

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1. Introduction

Strength and fracture parameters are very important in fracture mechanics. Hereupon when at crack tip exists stress concentration complex stress state is obtained. Here is not enough to determine highest stresses while strength and fracture are governed by equivalent effect. This effect also depends on material properties: elasticity and plasticity. In various studies [1-2] mostly brittle fracture is researched and plastic fracture studies are much more rare [3]. Fracture depend on crack size as well [4, 5]. Considering defect like round hole strength can be computed according theory of elasticity [6] and especially for fracture of brittle materials Griffiths criteria shall be applied [4]. But using this criteria it is complicated to determine crack growth and its area size. This task requires additional studies.

2. Brittle fracture criteria

Griffith’s energy growth criteria

\[ G = -\frac{\Delta \Pi}{\Delta S} > G_c \]  

where \( \Delta \Pi \) is system potential energy variation on increased crack surface \( \Delta S \); \( G_c \) is critical energy value.

However applying this criteria becomes hard to determine increase value of \( \Delta S \). Griffiths energy growth criteria is easier to calculate applying stress intensity coefficient \( K_I \). Then

\[ G = \frac{K_I^2}{E'} \]  

and

\[ E' = \begin{cases} E & \text{plane stress} \\ \frac{E}{1-\nu^2} & \text{plane strain} \end{cases} \]

where \( E \) is modulus of elasticity; \( \nu \) is Poisons’ ratio, \( K_I \) is stress intensity factor calculated on opening case (I mode).

Critical energy \( G_c \) value calculated by the equation

\[ G_c = \frac{K_{IC}^2}{E'} \]  

where \( K_{IC} \) is critical stress intensity factor.

Critical stress intensity factor \( K_{IC} \) is obtained by the equation

\[ K_{IC} = \sigma_\infty \sqrt{2\pi l_c} F_0 \]  

where \( \sigma_\infty \) is distant stresses, \( 2l_c \) is critical crack length, \( F \) is corrective function, taking into account crack and element geometry shown in Fig. 1.

![Fig. 1 Two opposite short cracks radiating from a circular hole in an infinite plate under tension](image)

For a plate with a far field uniform stress \( \sigma_\infty \) we know that there is a stress concentration factor of 3 [6]. For a crack radiating this hole we consider two cases. In one the crack is short \( l/2a \rightarrow 0 \) and thus we have an approximate for field stress of 3\( \sigma \) and for an edge crack \( F_1 = 1.12 \). Thus

\[ K_{IC} = 1.12(3\sigma_\infty)\sqrt{2\pi l_c} = 3.36\sigma_\infty \sqrt{2\pi l_c} \]  

In second case the crack is long \( 2a \leq 2l+2a \) and we can for all practical purposes ignore the presence of hole and assume that we have central crack with an effective length. Then

\[ l_{eff} = \frac{2l+2a}{2} = l + a \]
thus

$$K_{IC} = \sigma_c \sqrt{2\pi(a+l_c + a)}$$  \hspace{1cm} (7)

In studies [7] the plate with a hole and at its edge appeared crack $K_{IC}$ is obtained by the equation

$$K_{IC} = \sigma_c \sqrt{2\pi(a+l_c)}F$$  \hspace{1cm} (8)

where $a$ is radius of the hole.

Then

$$\sigma_c = \frac{K_{IC}}{\sqrt{2\pi(a+l_c)}F}.$$  \hspace{1cm} (9)

3. Multiparametric fracture criteria

McClintock and Leguilon [3, 4] offered strain fracture criteria when plastic zone at crack tip is taken into account and stresses $\sigma_c$ are calculated:

$$\sigma_c = \frac{2\sigma_c}{2 + \frac{a^2}{2(2a+l_c)} + 3\frac{a^4}{[2(2a+l_c)]^4}}$$  \hspace{1cm} (10)

where $\sigma_c$ is critical stresses at crack tip.

If no hole exists ($a = 0$), $\sigma_c = \sigma_c$.

When investigating maximum stresses at the crack tip $\sigma_{max}$ and crack tip area length $\Delta l_{max}$ in studies [5, 8] fracture resistance stresses $\sigma_{coh}$ were obtained

$$\sigma_{coh} = \sigma_{max} \left(1 - \frac{\Delta l}{\Delta l_{max}}\right)$$  \hspace{1cm} (11)

As a reference of studies [9]

$$G_c = \frac{1}{2} \Delta l_{max} \sigma_{max}$$  \hspace{1cm} (12)

In paper [10] an obtained criterion is described

$$G_c = h_c \gamma$$  \hspace{1cm} (13)

where $\gamma$ is fracture energy density (quantity of energy for unit volume), $h_c$ is critical crack area width dependable on stress concentration, $G_c$ is specific fracture energy.

With these principles three parameters criteria are formed [10]

$$G_c = \alpha G_c + (1-\alpha) G_c^u$$  \hspace{1cm} (14)

where $\alpha$ is the parameter indicating level of stress concentration ($0 < \alpha < 1$), $G_c$ is material fracture energy with existing crack, $G_c^u$ is fracture energy under tensile ultimate strength. Fracture energy is determined according strength and displacement tension diagram.

When fracture is specified by specific fracture energy then $\alpha = 1$, and when fracture is specified by stresses, then $\alpha = 0$.

4. Crack area studies

There are no analytic references in study [10] on how to specify crack area width $h_c$ and for parameter $\alpha$ assessment only the volume at crack tip analysis is proposed. But introduced analysis is approximate because complex stress and strain state exists. Therefore to determine crack area width would be the most proper by known stresses and fracture parameters equations [6]. Those equations for plane stress are

$$\alpha = \frac{r_c}{r_a}$$  \hspace{1cm} (21)

when $r_c = 0$, $\alpha = 0$ fracture with no crack exists.

Taking into account that $\theta = 0$, radius $r_a$ for plane stress calculated by Eq. (16)
For plane strain

\[ r_c = \frac{K_{IC}^2}{\sigma_{IC}^2} \frac{2}{6\pi} \]  

(22)

For plane strain

\[ r_c = \frac{K_{IC}^2}{\sigma_{IC}^2} \frac{3}{6\pi} \]  

(23)

Then three parameters criteria (14) can be obtained taking into account:
1) \( G_c \) which is calculated from Eq. (3);
2) fracture energy \( G^u_c \) obtained by plate tension with no crack and hole in \( F-\Delta u \) coordinates till ultimate strength;
3) parameter \( \alpha \) which is obtained from Eq. (21).

5. Experiment

For the experiment three different steel grade (Table 1) specimens were chosen: 1) Steel 45, 2) Steel 15XCHNД, 3) Steel 35ХГСА.

Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield strength ( \sigma_Y ), MPa</th>
<th>Ultimate strength ( \sigma_U ), MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel 45</td>
<td>320</td>
<td>680</td>
</tr>
<tr>
<td>Steel 15XCHNД</td>
<td>350</td>
<td>630</td>
</tr>
<tr>
<td>Steel 35ХГСА</td>
<td>404</td>
<td>730</td>
</tr>
</tbody>
</table>

Round hole was drilled through the plate and cracks were made inside the hole with the help of laser cuts.

Energy \( G^u_c \) was calculated and expressed by tension diagram area till ultimate strength. For the plane stress plate dimensions – 2x50 mm and diameter of holes – 2; 5 mm. When 2 mm plate is broken by tension force no significant plate cross-section shrinkage was noticed. That’s why this deformation is considered as a plain strain.

Fracture characteristics \( \sigma_Y, \sigma_U, l, \) and \( K_{IC} \) were obtained during the experiment and calculative area \( r_c \) is shown in Table 2. Calculative fracture parameters values \( r_c, \alpha, G_c, G^u_c \) and \( G^S_c \) are shown in Table 3.

Table 2

<table>
<thead>
<tr>
<th>No.</th>
<th>( a ), mm</th>
<th>( 2l ), mm</th>
<th>( \sigma_Y ), MPa</th>
<th>( \sigma_U ), MPa</th>
<th>( r_c ), mm</th>
<th>( r_c ), mm</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>424</td>
<td>1427</td>
<td>2</td>
<td>0.5</td>
<td>0.250</td>
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<tr>
<td>2</td>
<td>1.3</td>
<td>2</td>
<td>372</td>
<td>1251</td>
<td>2</td>
<td>0.65</td>
<td>0.17</td>
</tr>
<tr>
<td>3</td>
<td>1.6</td>
<td>3</td>
<td>440</td>
<td>1480</td>
<td>2</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
<td>394</td>
<td>1324</td>
<td>3.8</td>
<td>1</td>
<td>0.26</td>
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Steel 15XCHNД

<table>
<thead>
<tr>
<th>No.</th>
<th>( a ), mm</th>
<th>( 2l ), mm</th>
<th>( \sigma_Y ), MPa</th>
<th>( \sigma_U ), MPa</th>
<th>( r_c ), mm</th>
<th>( r_c ), mm</th>
<th>( \alpha )</th>
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<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>490</td>
<td>1648</td>
<td>2</td>
<td>1.5</td>
<td>0.75</td>
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<tr>
<td>6</td>
<td>3.7</td>
<td>5</td>
<td>441</td>
<td>1484</td>
<td>3.8</td>
<td>1.85</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Steel 35ХГСА

Obtained fracture energy results show reliance on size of hole and crack relation and deformation state.

For the plain strain, plate dimensions – 4x50 mm and diameter of holes – 2; 5 mm. When 4 mm plate is broken by tension force, plate cross-section shrinkage is noticed. That’s why this deformation is considered as a plain strain.

Fracture characteristics \( \sigma_Y, \sigma_U, l, \) and \( K_{IC} \) were obtained during the experiment and calculative area \( r_c \) is shown in Table 4. Calculative fracture parameters values \( r_c, \alpha, G_c, G^u_c \) and \( G^S_c \) are shown in Table 5.

Table 3

<table>
<thead>
<tr>
<th>No.</th>
<th>( K_{IC} ), MPa·m(^{3/2} )</th>
<th>( G_c ), kJ/m(^2 )</th>
<th>( G^u_c ), kJ/m(^2 )</th>
<th>( G^S_c ), kJ/m(^2 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>32.0</td>
<td>4.84</td>
<td>11.63</td>
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<tr>
<td>2</td>
<td>105</td>
<td>55.1</td>
<td>8.03</td>
<td>26.9</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
<td>128.0</td>
<td>7.05</td>
<td>97.8</td>
</tr>
</tbody>
</table>

Value of energy and fracture parameters under plane strain

Table 4

<table>
<thead>
<tr>
<th>No.</th>
<th>( a ), mm</th>
<th>( 2l ), mm</th>
<th>( \sigma_Y ), MPa</th>
<th>( \sigma_U ), MPa</th>
<th>( r_c ), mm</th>
<th>( r_c ), mm</th>
<th>( \alpha )</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>2</td>
<td>0.8</td>
<td>450</td>
<td>1512</td>
<td>2</td>
<td>0.37</td>
<td>0.18</td>
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<tr>
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<td>1.1</td>
<td>420</td>
<td>1411</td>
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<td>0.42</td>
<td>0.11</td>
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<tr>
<td>3</td>
<td>2</td>
<td>1.4</td>
<td>480</td>
<td>1612</td>
<td>2</td>
<td>0.56</td>
<td>0.28</td>
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<tr>
<td>4</td>
<td>5</td>
<td>1.8</td>
<td>440</td>
<td>11478</td>
<td>3.8</td>
<td>0.66</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Steel 15XCHNД

<table>
<thead>
<tr>
<th>No.</th>
<th>( a ), mm</th>
<th>( 2l ), mm</th>
<th>( \sigma_Y ), MPa</th>
<th>( \sigma_U ), MPa</th>
<th>( r_c ), mm</th>
<th>( r_c ), mm</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>2.5</td>
<td>520</td>
<td>1747</td>
<td>2</td>
<td>0.115</td>
<td>0.057</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>3.1</td>
<td>460</td>
<td>1545</td>
<td>3.8</td>
<td>0.146</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Steel 35ХГСА

Table 5

<table>
<thead>
<tr>
<th>No.</th>
<th>( K_{IC} ), MPa·m(^{3/2} )</th>
<th>( G_c ), kJ/m(^2 )</th>
<th>( G^u_c ), kJ/m(^2 )</th>
<th>( G^S_c ), kJ/m(^2 )</th>
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<tbody>
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<td>1</td>
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<td>29.1</td>
<td>4.4</td>
<td>8.94</td>
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<td>2</td>
<td>95.5</td>
<td>50.1</td>
<td>7.3</td>
<td>19.28</td>
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<td>3</td>
<td>145.6</td>
<td>116.5</td>
<td>6.4</td>
<td>12.67</td>
</tr>
</tbody>
</table>

Steel 35ХГСА

Obtained fracture energy results show reliance on size of hole and crack relation and deformation state.
6. Conclusions

1. Fracture analysis indicates that structural elements made of plastic and semiplastic materials with stress concentration around holes edges generate plastic deformations and fracture with crack growth. For strength and fracture assessment is necessity to apply multiparametric fracture criteria.

2. As yet two parameters fracture criteria were used, therefore three parameters criteria describes fracture process in more details. Those parameters are: material fracture energy with existing crack, fracture energy obtained when ultimate strength is reached and parameter indicating stress concentration level.

3. Evaluating crack area width and deformation width of stress concentrator edges, similarly calculated fracture energy with and without crack – certain specific fracture energy calculations are provided. Strength and fracture characteristics determined from experimental data by testing plates with holes specimens.

References


A. Žiliukas, A. Surantas, G. Žiogas

STIPRUMO IR IRIMO KRITERIJŲ TAIKYMAS ĮTEMPIŲ KONCENTRATORIŲ ZONOSE

R e z i u m e

Straipsnyje analizuojamas pagamintų plokštelėų su skylėmis, iš plastinių plienų, plastinis devomavimas skylės kraštuose ir irimais atsiradus plyšiui. Stiprumui ir irimui įvertinti apskaičiuojama specifinė irimo energija, gaunama kaip plyšio irimo energijos ir deformacijos energijos dydžio, esant stiprumo ribai, suma, susieta parametrui, parodančiu įtempių koncentracijos lygį. Skaiciavimams naudojamos bandymų metu nustatyto stiprumo ir irimo charakteristikos.

A. Žiliukas, A. Surantas, G. Žiogas

STRENGTH AND FRACTURE CRITERIA APPLICATION IN STRESS CONCENTRATORS AREAS

S u m m a r y

This paper presents an investigation of plastic deformation of plates with holes made of plastic steels. Plastic deformation in plate hole edge area and fracture with appeared crack are studied. Specific fracture energy is calculated for strength and fracture evaluation which is obtained as a sum of fracture energy crack and deformation energy value with ultimate strength limit coherent with the parameter, describing stress concentration level. For the calculations strength and fracture parameters are used which were obtained during the experiment.

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ИСПОЛЬЗОВАНИЕ ПАРАМЕТРОВ ПРОЧНОСТИ И РАЗРУШЕНИЯ В ЗОНАХ КОНЦЕНТРАЦИИ НАПРЯЖЕНИЙ

Резюме

В настоящей работе проведено исследование пластического деформирования пластин с отверстиями из пластических сталей. Рассматривается пластическое деформирование зон около отверстий пластин и их разрушение. Рассчитана относительная суммарная энергия деформирования и разрушения при предельных состояниях в зависимости от уровня концентрации напряжений. Расчётные параметры упругости и разрушения получены экспериментально.

Received March 05, 2010
Accepted June 03, 2010