Low cycle stress strain curves and fatigue under tension-compression and torsion

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1. Introduction

During exploitation damage gradually appears in the constructions materials and results their fracture. The gradually accumulated damage depends on material properties, magnitude and character of the time-dependent stress and strain variation, environment conditions. It was observed, that 75% of fracture in mechanical constructions is causes by the material fatigue. Especially dangerous are the overloads, as cyclically varying loading exceeds the proportionality limit of the material and causes plastic strain and formation of the hysteresis loop, while durability of the material decreases to thousands or hundreds of cycles [1]. In most mechanisms and devices under loading the elastic-plastic strain appears in stress concentration areas, near the sudden change of the shape, e.g. in key seats, near shafts diameter changing places, as a result of incorrectly chosen fillet radius, in welded joints, because of the various welding defects and etc. [2, 3]. Under cyclic elastic-plastic loading, after the cycle number of hundreds – thousands, the fatigue crack appears which commonly causes failures with hardly predictable outcome.

The problems of metal fracture remain actual despite years of long-lasting investigation of the cyclic loading of metals [4]. While selecting the material, it is necessary to know properties and change laws of their characteristics under different type loading in the areas of the periodically varying elastic-plastic strain. Most common are the following three types of loading: tension-compression, bending and torsion [5].

If compared to tension-compression and bending tests, the number of performed low cycle torsion loading tests is not so considerable. It should be noted, that a large amount of the parts in real operating conditions, i.e. shafts, springs and others parts of the mechanisms, are exactly under cyclically varying torsion loading [6, 7].

2. Experimental setup and used specimens

All performed experimental analyses: monotonous tension, monotonous torsion, low cycle tension-compression and low cycle torsion were carried out under ambient temperature. For both the mentioned cases, the specimens were under symmetric loading and experimental data was registered up to crack initiation.

For monotonous tension and low cycle tension-compression fatigue analysis the experimental low cycle setup with $T=500$ Nm torque and the same electronic equipment, as in tension-compression analysis, was used.

Tubular shape specimens with $t/d=1/20$ working part were used for the experiments. The specimen is shown in Fig. 2. During the cyclic torsion uniform stress state is produced within the wall of the tubular specimen, i.e. the stress gradient does not have the influence. To fulfill the working part of the test, the same fillet radius $R=25$ mm was used for both the torsion and tension-compression specimens, aiming to decrease the stress concentration to minimum (the theoretical stress concentration coefficient $\alpha_s \approx 1.03$).

To determine the torque $T$, resistance wire gauges were glued on the surface of the device with cylindrical working part $d=18.0$ mm. This device is made of the thermal treated grade 60S2A spring steel (HRC 42-45). The working strain gauges were glued to the cylinder’s surface along the main strain directions $e_1$ and $e_3$ (at 45° angle, in opposite sides).
The torsion strain is measured by the attachment, which identifies the torsion angle $\varphi$ in the working part of the specimen. The device for torsion angle measurements, presented in Fig. 3, consists of two rings 1 and 2, each of them has bolt fastened half rings, that are attached to the specimen by means of the 4 conical tip bolts, locating them at identical angles. Two spring steel plates 3 and 4 are fastened to the top ring. Working gauges ($R=100\,\Omega$) are glued along tension-compression sides of the plates. Free end of each plate rests on bolt-adjusted bottom retainer ring. During torsion of the specimen, the rings turn relative to each other and sprung steel plates act as cantilever rods during bending.

3. Experimental analysis

3.1. Investigation of the monotonous loading

During the experiments of monotonous loading, the monotonous tension and monotonous torsion curves were obtained. The curves of the monotonous tension and torsion in coordinates $\sigma_i - e_i$, and $\tau_{max} - \gamma_{max}$ are presented in Figs. 4 and 5. The determined mechanical characteristics of the grade 45 steel under tension are given in Table 1 and under torsion – in Table 2.

The curves of monotonous tension in $\sigma_{i} - e_{i}$ coordinates were obtained applying the equalities

$$\sigma_{i} = \sigma_{1} \; ; \; e_{i} = e_{1}$$  \hspace{1cm} (1)

Fig. 3 Tenzometer for torsion angle measurements

The curves of monotonous torsion in $\tau_{i} - e_{i}$ coordinates were obtained by the Eqs. 2.

$$\sigma_{i} = \sqrt{3}\tau ; \; e_{i} = \frac{\gamma}{\sqrt{3}}$$  \hspace{1cm} (2)

Fig. 4 Curves of the monotonous tension and torsion

The curves of monotonous tension in $\tau_{max} - \gamma_{max}$ coordinates were obtained by

$$\tau_{max} = \frac{\sigma_{1}}{2} \; ; \; \gamma_{max} = 1.5e_{1}$$  \hspace{1cm} (3)

Fig. 5 Curves of monotonous tension and torsion

The curves of monotonous torsion in $\tau_{max} - \gamma_{max}$ coordinates were obtained by the Eq. 4.

$$\tau_{max} = \tau ; \; \gamma_{max} = \gamma$$  \hspace{1cm} (4)

It is seen from the Figs. 4 and 5 that monotonous tension and torsion curves in $\sigma_{i} - e_{i}$ coordinates are closer than the same curves in $\tau_{max} - \gamma_{max}$ coordinates.

Table 1

<table>
<thead>
<tr>
<th>$\sigma_{0i}$, MPa</th>
<th>$\sigma_{02i}$, MPa</th>
<th>$\sigma_{03i}$, MPa</th>
<th>$\sigma_{04i}$, MPa</th>
<th>$\sigma_{u2i}$, %</th>
<th>$\psi_{02i}$, %</th>
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<td>324</td>
<td>328</td>
<td>688</td>
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<td>13.0</td>
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Table 2

<table>
<thead>
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<th>$\tau_{0i}$, MPa</th>
<th>$\tau_{02i}$, MPa</th>
<th>$\tau_{03i}$, MPa</th>
<th>$\tau_{04i}$, MPa</th>
<th>$\gamma_{02i}$, %</th>
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</thead>
<tbody>
<tr>
<td>174</td>
<td>226</td>
<td>425</td>
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<td>224</td>
<td>209</td>
<td>435</td>
<td>25.2</td>
<td></td>
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<td>188</td>
<td>211</td>
<td>420</td>
<td>19.7</td>
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</tbody>
</table>

3.2. Low cycle stress strain curves

Under stress limited low cycle loading, the determined hysteresis loop width dependence both on the number of loading semicycles $k$ and loading level $\bar{\sigma}$, is presented in Fig. 6. The mentioned data was obtained during the tension-compression experiments, using loading levels from $\bar{\sigma} = 1.08$ to $\bar{\sigma} = 1.93$, where
\[ \sigma_0 = \frac{\sigma}{\sigma_{pl}}; \quad \delta_k = \frac{\delta_k}{\varepsilon_{pl}} \]  

(5)

Here \( \sigma_0 \) is loading stress amplitude, \( \delta_k \) is width of the hysteresis loop of plastic strain for loading semicycle \( k \), \( \sigma_{pl} \) and \( \varepsilon_{pl} \) are stress and strain of proportionality limit [1].

\[ \delta_k = A_{1,2} \left( \frac{\varepsilon_0 - \frac{\varepsilon_f}{2}}{\varepsilon_0} \right)^k \]  

(6)

where \( A_1 \), \( A_2 \) and \( \alpha \) are cyclic characteristics of the material, \( \varepsilon_0 \) is relative initial strain, \( \varepsilon_f \) is cyclic proportionality limit.

To determine tension-compression constants \( A_1 \) and \( A_2 \), and torsion constant \( A \) under stress limited low cycle loading, \( \delta_{1,2} = f(\varepsilon_k) \) graphs of the semicycle hysteresis loop width dependence on initial strain have been used [1], i.e.

\[ A_{1,2} = \frac{\delta_{1,2}}{\delta_0} \]  

(7)

Dependences of semicycle’s loop width on the initial strain are shown in Figs. 8 and 9, whereas determined cyclic characteristics of the material are given in Table 3.

| Cyclic characteristics of the grade 45 steel |
|-----------------|-----------------|-----------------|-----------------|
| Tension-compression | Torsion | |
| \( A_1 \) | \( A_2 \) | \( \frac{\varepsilon_f}{2} \) | \( \alpha \) | \( A \) | \( \frac{\varepsilon_f}{2} \) | \( \alpha \) |
| Grade 45 steel | |
| 0.93 | 1.01 | 1.65 | 0 | 1.14 | 1.40 | 0 |

Carrying out the low cycle tension-compression tests, it was obtained, that grade 45 steel is accumulating
plastic strain in tension direction (Fig. 10). Thus, the accumulated plastic strain after loading semicycles $k$, can be expressed as follows [1]

$$\bar{\varepsilon}_{pk} = \bar{\varepsilon}_0 - \bar{\sigma}_0 + \sum_{i=1}^{k} (-1)^i \bar{\delta}_k$$

(8)

Carrying out the low cycle torsion tests, it was obtained, that grade 45 steel does not accumulate plastic strain.

3.2. Low cycle fatigue curves

Fig. 11 presents curves of low cycle fatigue and reduction of area $\psi$ for grade 45 steel under stress limited tension-compression loading.

Fig. 12 presents low cycle fatigue curves of grade 45 steel under strain limited torsion: 1 – in coordinates $\lg \bar{\sigma} - \lg k_c$; 2 – in coordinates $\lg \bar{\varepsilon} - \lg k_c$.

Fig. 13 Fatigue curves of grade 45 steel under strain limited tension-compression: 1 – in coordinates $\lg \bar{\delta} - \lg k_c$; 2 – in coordinates $\lg \bar{\varepsilon} - \lg k_c$.

Fig. 14 Fatigue curves of grade 45 steel under strain limited torsion: 1 – in coordinates $\lg \bar{\delta} - \lg k_c$; 2 – in coordinates $\lg \bar{\varepsilon} - \lg k_c$.

<table>
<thead>
<tr>
<th>Values of Coffins constants $C$ and $m$</th>
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<tr>
<td>Steel 45</td>
</tr>
<tr>
<td>$C_1$</td>
</tr>
<tr>
<td>314</td>
</tr>
<tr>
<td>727</td>
</tr>
</tbody>
</table>

4. Damage under low cycle loading

Under tension-compression and stress limited loading, fracture of the specimen occurs due to quasistatic
damage \( d_{K} \), caused by the accumulated plastic strain \( \overline{\epsilon}_{pk} \), and fatigue damage \( d_{N} \), caused by the cyclic plastic strain, which is caused by the hysteresis loop width \( \delta_{k} \), whereas total damage may be written [1]

\[
d = d_{K} + d_{N} \tag{9}
\]

where \( d \) is total damage.

Fatigue damage is calculated by the equation

\[
d_{N} = \frac{k}{k} \sum \delta_{k} \tag{10}
\]

where \( \sum \delta_{k} \) is fatigue damage accumulated during the \( k \) loading semicycles, \( \sum \delta_{k} \) is fatigue damage accumulated till crack initiation.

Quasistatic damage

\[
d_{K} = \frac{\overline{\epsilon}_{pl}}{\overline{\epsilon}_{u2}} \tag{11}
\]

where \( \overline{\epsilon}_{pl} \) is accumulated plastic strain during \( k \) semicycles of loading, whereas \( \overline{\epsilon}_{u2} \) is the maximum uniform strain under monotonous loading which corresponds \( \sigma_{u} \).

If stress limited loading is approached as non-stationary strain limited loading, when the damage, accumulated during one semicycle \( k \), is expressed as

\[
d_{K} = \frac{\overline{\epsilon}_{k}}{\sum \delta_{k}} \tag{12}
\]

Then condition of the crack initiation is

\[
\frac{\overline{\delta}_{k}}{\sum \delta_{k}} + \frac{\overline{\delta}_{k}}{\sum \delta_{k}} + \ldots + \frac{\overline{\delta}_{k}}{\sum \delta_{k}} = 1 \tag{13}
\]

The analysis of strain limited low cycle loading when strain is limited and quasistatic damage is not occurring was performed. In this case, damage of the specimen is predetermined only by the cyclic plastic strain, i.e. under strain limited loading, the fatigue curve in coordinates \( \lg \overline{\delta} - \lg k_{c} \) has a shape of straight line. The constants \( m \) and \( C \) have been determined by the equation of straight line

\[
\overline{\delta} = -m \lg k_{c} + \lg C \tag{14}
\]

or

\[
\overline{\delta} k_{c}^{m} = C \tag{15}
\]

where \( \overline{\delta} \) is average width of the hysteresis loop.

From the fatigue curve, formed under strain limited loading, in coordinates \( \lg \overline{\delta} - \lg k_{c} \) and \( \lg \overline{\delta} - \lg k_{c} \) and applying the L.Coffin’s equation

\[
\overline{\delta}_{m} = C_{2} k_{c}^{-m_{2}} \tag{16}
\]

In expression (16), the average width of the plastic hysteresis loop was calculated by the equation:

\[
\overline{\delta}_{m} = \frac{1}{k_{c}} \sum \delta_{k} \tag{17}
\]

therefore

\[
\sum \delta_{k} = C_{2} k_{c}^{1-m_{2}} \tag{18}
\]

Applying the coordinates \( \lg \overline{\delta} - \lg k_{c} \), we obtain

\[
\overline{\delta} k_{c}^{m_{2}} = C_{3} \tag{19}
\]

and

\[
k_{c} = \frac{C_{3}}{m_{2}} \tag{20}
\]

After the applied Eq. (20)

\[
\sum \delta_{k} = C_{2} \frac{C_{3}^{1-m_{2}/m_{2}}}{C_{3}^{1-m_{2}/m_{2}}} \tag{21}
\]

By introducing the \( m_{1} = 1-m_{2}/m_{2} \) and applying the Eqs. (13) - (21), we obtain

\[
\frac{\overline{\delta}_{k} \overline{\epsilon}_{k}^{m_{2}}}{C_{2} C_{3}^{1-m_{2}/m_{2}}} + \frac{\overline{\delta}_{k} \overline{\epsilon}_{k}^{m_{2}}}{C_{2} C_{3}^{1-m_{2}/m_{2}}} + \ldots + \frac{\overline{\delta}_{k} \overline{\epsilon}_{k}^{m_{2}}}{C_{2} C_{3}^{1-m_{2}/m_{2}}} = 1 \tag{22}
\]

Because of good agreement between the experimental and calculated data, Eq. (22) was used to calculate the damage in works [8, 9].

Fig. 15 Curves of low cycle fatigue under stress limited loading, respectively: 1 – tension-compression, 2 – torsion, 3 – theoretical curve of the tension-compression, when only fatigue damage is taken into account, 4 – theoretical curve of the torsion, as only fatigue damage is taken into account.
The curve 3 in Fig. 15 presents the fatigue damage under stress limited tension-compression as only fatigue damage is taken into account and is close to the fatigue curve under stress limited torsion (curve 2), because under stress limited torsion loading, strain accumulation is not observed, i.e., the quasistatic damage does not occur.

The curve 4 confirms Eq. (22), as it shows satisfactory agreement to fatigue curve under stress limited torsion, because during the torsion experiments the quasistatic damage was not observed. The curves 3 and 4 confirm that according to the results of cyclic tension-compression it is possible to calculate the durability under cyclic torsion. Besides, the durability under cyclic torsion (in coordinates $\sigma_i - N_i$) is higher than under cyclic tension-compression loading, because under cyclic torsion there is no accumulation of plastic strain, i.e. there is no quasistatic damage.

5. Conclusions

Grade 45 steel was investigated under monotonous tension, monotonous torsion, the low cycle tension-compression and low cycle torsion with stress and strain limited loading, using the circular cross-section specimens for tension-compression and thin walled specimens – for torsion.

1. It was determined, that characteristics of the cyclic stress strain curves for the analyzed grade 45 steel at cyclic tension-compression, and under cyclic torsion, are similar. For both the analyzed loading cases the parameter $\alpha = 0$, i.e. the material is cyclically stable. Values of the parameters $A$, which characterizes the hysteresis loop width of the first semicycle, are also similar.

2. During the stress limited loading, under cyclic torsion, the accumulation of plastic strain was not observed, i.e. under stress limited torsion, there is no quasistatic damage.

3. The durability under cyclic torsion is higher (in coordinates $\sigma_i - N_i$), than that under cyclic tension - compression loading, because under cyclic torsion there is no accumulation of plastic strain, i.e. is no quasistatic damage.

References


M. Daunys, R. Česnapičius

MAŽACIKLIO DEFORMAVIMO IR SUIRIMO KREIVĖS ESANT TEMPIMUI-GNIUŽDYMUI IR SUKIMUI

Reziumė

Straipsnyje nagrinėjamas plieno 45 mažaciklis nuovargis esant apribotiemis įtempiams ir deformacijoms bei tempimui-gniuždymui ir sukimu.

Nustatyta, kad tiriamojo medžiaga yra cikliškai stabili tiek tempiai bei gniuždoma, tiek sukima (parametras $\alpha = 0$), o deformavimo diagramų forma abiem apkrovinio atvejais yra panaši, nes parametrai $A$ yra panašūs tiek esant tempimui bei gniuždymui, tiek sukimu. Ciklinio sukimo metu, esant apkrovinui su apribotais įtempiais, vienpusė plastinė deformacija nekaupiamas, t. y. nėra kvazistatinių pažeidimų. Straipsnyje panaudotos analitinės priklausomybės nuovargio pažeidimams apskaičiuoti gerai tenkina ciklinio sukimo duomenis, nes teorinė ciklinio sukimo nuovargio kreivė gerai sutampa su eksperimentinė kreivė, gauta esant apribojus įtempiams, ir yra artima teorinei ciklinio tempimo bei gniuždymo nuovargio kreivei įvertinant tik nuovargio pažeidimus esant apkrovui su apribotais įtempiais.

Tai rodo, kad, esant cikliniam sukimum, ilgalaikškumas (koordinatėse $\sigma_i - N_i$) yra didesnis už ilgalaikškumą esant cikliniam tempimui bei gniuždymui, nes šiuo atveju plastinė deformacija nekaupiamas t. y. nėra kvazistatinių pažeidimų.

M. Daunys, R. Česnapičius

LOW CYCLE STRESS STRAIN CURVES AND FATIGUE UNDER TENSION-COMPRESSION AND TORSION

Summary

The presented paper analyses low cycle fatigue of the grade 45 steel under stress and strain controlled low cycle tension-compression and torsion.

It was determined, that analysed material is cyclically stable, in both the tension-compression and torsion cases, because the parameter $\alpha = 0$. The shape of the stress strain diagrams for both the analysed loading cases is similar, as parameters $A$ are similar for the tension-compression and the torsion. During the cyclic torsion, under stress limited loading, the accumulation of plastic strain is not ob-
served, i.e. quasistatic damage does not occur. The analytical dependences applied for the fatigue damage calculation showed good agreement with the cyclic torsion data, since the theoretical fatigue curve for cyclic torsion showed good agreement with the experimental curve for the stress limited case – and is close to the theoretical fatigue curve under stress limited cyclic tension-compression loading, when estimating only fatigue damage.

This shows, that under cyclic torsion, the durability (in $\sigma_i - N_c$ coordinates) is higher than the durability under cyclic tension-compression, because in this case the plastic strain is not accumulated, i.e. the quasistatic damage does not occur.

М. Даунис, Р. Чеснавичюс

КРИВЫЕ МАЛОЦИКЛОВОГО ДЕФОРМИРОВАНИЯ И РАЗРУШЕНИЯ ПРИ РАСТЯЖЕНИИ-СЖАТИИ И КРУЧЕНИИ

Резюме

В статье представлен анализ результатов малоцикловой усталости стали 45 при ограниченных напряжениях и деформациях, в условиях растяжения-сжатия и кручения.

Установлено, что исследованный материал является циклически стабильным при растяжении-сжатии и кручении (параметр $\alpha = 0$) и для обоих случаев нагрузки диаграммы деформирования близки по форме, в связи с тем, что значения параметров $A$ для обоих случаев нагрузки отличаются мало. При циклическом кручении с ограниченным моментом кручения отсутствует накопление односторонней пластической деформации, т. е. нет квазистатических повреждений. Аналитические зависимости, использованные для расчета усталостных повреждений, дали хорошую сходимость с данными циклического кручения, так как теоретическая усталостная кривая хорошо сходится с экспериментальной кривой кручения при ограниченных напряжениях и близка к теоретической усталостной кривой при циклическом растяжении-сжатии, когда при нагрузке с ограниченными напряжениями учитываются только усталостные повреждения.

Полученные результаты показывают, что долговечность (в координатах $\sigma_i - N_c$) при циклическом кручении больше, чем долговечность при циклическом растяжении-сжатии, так как в случае кручения отсутствует накопление пластической деформации, т. е. нет квазистатических повреждений.

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