Magnetic fluid based squeeze film between curved rough circular plates

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Nomenclature

- $a$ - radius of the circular plate; $p$ - lubricant pressure; $B$ - curvature parameter of the upper plate; $C$ - curvature parameter of the lower plate; $H$ - magnitude of the magnetic field; $p = -\frac{h_0^2 p}{\mu h_0 a^2}$ - dimensionless pressure; $W$ - load carrying capacity; \[ W = -\frac{h_0^2 W}{\mu h_0 a^2} \] - dimensionless load carrying capacity; $\Delta t$ - response time; $\Delta T = \frac{\Delta t Wh_0^2}{\pi \mu a^2}$ - non-dimensional response time; $\alpha$ - mean of the stochastic film thickness; $\sigma$ - standard deviation of the stochastic film thickness; $\varepsilon$ - measure of symmetry of the stochastic random variable; $\sigma = \sigma/h_0$; $\alpha = \alpha/h_0$; $\varepsilon = \varepsilon / h_0^2$; $R = r / a$; $\varphi$ - inclination angle; $\mu$ - absolute viscosity of the lubricant; $\mu$ - magnetic susceptibility; $\mu_0$ - permeability of the free space; $\mu_0 = \frac{-\mu_0 \phi h^3}{\mu h_0}$ - magnetization parameter.

1. Introduction

F.R. Archibald [1] discussed the behavior of squeeze film between various geometrical configurations of flat surfaces. D.F. Hays [2] presented the squeeze film phenomena between curved plates considering curvature of the sine form and keeping minimum film thickness as constant. P.R.K. Murty [3] analyzed the behavior of squeeze film trapped between curved circular plates describing the film thickness by an expression of an exponential function. He based his analysis on the assumption that the central film thickness of minimum film thickness as assumed by D.F. Hays, was kept constant. It was established that the load carrying capacity rose sharply with curvature parameter of the upper plate and the dimensionless load carrying capacity $\Delta T$, for a magnetic fluid based squeeze film between curved circular plates. The magnetic fluid based squeeze film between curved surfaces determines the performance of the squeeze film. However, here the plates were taken to be flat. But in actual practice the flatness of the plate does not endure owing to elastic, thermal and uneven wear effects. With this end in view M.V. Bhat and G.M. Deheri [10], where in, they concluded that the application of magnetic fluid lubricant improved the performance of the squeeze film. However, the application of magnetic fluid lubricant improved the performance of the squeeze film. It is a well-known fact that after having some run-in and wear the bearing surfaces develop roughness. The roughness often appears random and disordered and does not seem to follow any particular structural pattern. The randomness and the multiple roughness scales both contribute to the complexity of the surface geometrical structure. Invariably, it is this complexity which contributes to most of the problems in studying friction and wear. The random character of the surface roughness was recognized by several investigators who employed a stochastic approach to mathematically model the roughness of the bearing surfaces (S.T. Tzeng and E. Seibel [14], H. Christensen and K.C. Tonder [15 - 17]). K.C. Tonder [18] analyzed theoretically the transition between surface distributed waviness and random roughness. S.T. Tzeng and E. Seibel [14] used a beta probability density function for the random variable characterizing the roughness. This distribution is symmetrical in nature with zero mean and approximate the Gaussian distribution to a good degree of accuracy for certain special cases. H. Christensen and K.C. Tonder [15 - 17] further developed and modified this approach and proposed a comprehensive general analysis both for transverse as well as longitudinal surface roughness based on a general probability density function. H. Christensen and K.C. Tonders method developed the framework to study the effect of surface roughness on the performance of bearing system in a number of investiga-
tions. (L.L Ting [19], J. Prakash and K. Tiwari [20], B.L. Prajapati [21], S.K. Guha [22], J.L. Gupta and G.M. Deheri [23]). In all these analysis the probability density function for the random variable characterizing the surface roughness was assumed to be symmetric with mean of the random variable equal to zero. However, in general, this may only be true to the first approximation. In practice, due to nonuniform rubbing of the surfaces the distribution of surface roughness may indeed be asymmetrical. Thus, with this idea in view, P.I. Andharia, J.L. Gupta and G.M. Deheri [24] discussed the effect of transverse surface roughness on the performance of a hydrodynamic squeeze film in a spherical bearing using the general stochastic analysis. It was observed that the effect of transverse surface roughness on the performance of the bearing was considerably adverse.

Here we propose to analyze magnetic fluid based squeeze film between two curved transversely rough circular plates, where in, the upper plate lies along the surface determined by hyperbolic function while, the lower plate lies along the surface governed by secant function.

2. Analysis

Configuration of the bearing is displayed in Fig. 1.

![Fig. 1 Bearing configuration](image)

The bearing surfaces are considered to be transversely rough. The thickness \( h(x) \) of the lubricant film is

\[
h(x) = \overline{h}(x) + h_s
\]

where \( \overline{h}(x) \) is the mean film thickness while \( h_s \) is the deviation form the mean film thickness characterizing the random roughness of the bearing surfaces. The deviation \( h_s \) is assumed to be stochastic in nature and described by the probability density function

\[
f(h_s) - c \leq h_s \leq c
\]

where \( c \) is the maximum deviation from the mean film thickness. The mean \( \alpha \), the standard deviation \( \sigma \) and the parameter \( \varepsilon \) which is the measure of symmetry associated with random variable \( h_s \) are governed by the relations

\[
\alpha = E(h_s)
\]

and

\[
\varepsilon = E\left(\frac{h_s - \alpha}{\sigma}\right)^3
\]

where \( E \) denotes the expected value defined by

\[
E(R) = \int_{-\infty}^{\infty} Rf(h_s)dh_s
\]

We assume the upper plate lying along the surface determined by

\[
Z_u = h_0 \left[ \frac{1}{1 + Br} \right]; \quad 0 \leq r \leq a
\]

approaching with normal velocity \( h_0 = \frac{dh_0}{dt} \), to the lower plate lying along the surface

\[
Z_l = h_0 \left[ \sec(-Cr^2) - 1 \right]; \quad 0 \leq r \leq a
\]

where \( h_0 \) is the central distance between the plates, \( B \) and \( C \) are the curvature parameters of the corresponding plates. The central film thickness \( h(r) \) then is defined by

\[
h(r) = h_0 \left[ \frac{1}{1 + Br} - \sec\left(-Cr^2\right) + 1 \right]
\]

Axially symmetric flow of the magnetic fluid between the plates is taken into consideration under an oblique magnetic field

\[
\overline{H} = (H(r)\cos\phi(r,z), 0, H(r)\sin\phi(r,z))
\]

whose magnitude \( H \) vanishes at \( r = a \); for instance; \( H^2 = k(a-r), 0 \leq r \leq a \), where \( k \) is a suitably chosen constant so as to have a magnetic field of required strength, which suits the dimensions of both the sides. The direction of the magnetic field plays a significant role since \( \overline{H} \) has to satisfy the equation

\[
\nabla \times \overline{H} = 0; \quad \nabla \times \nabla \times \overline{H} = 0
\]

Therefore, \( \overline{H} \) arises out of a potential function and the inclination angle \( \phi \) of the magnetic field \( \overline{H} \) with the lower plate is determined by

\[
cot\phi \frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial z} = \frac{1}{2(a-r)}
\]

whose solution is determined from the equations

\[
c_i^2 \csc^2 \phi = a - r; \quad z = -2c_i \sqrt{(a-c_i^2 - r)}
\]
where \( c \) is a constant of integration.

The modified Reynolds equation governing the film pressure \( p \) can be obtained as [12, 23, 25]

\[
\frac{1}{r} \frac{d}{dr} \left[ rg(h) \frac{d}{dr} \left( p - 0.5 \mu_0 \mu H^2 \right) \right] = 12 \mu \dot{h}_0
\]  

(14)

where

\[
g(h) = h^3 + 3\sigma h + 3\alpha^2 + 3\alpha^3 + \epsilon
\]  

(15)

Introducing the nondimensional quantities

\[
\bar{h} = \frac{h}{h_0}; \quad \mu = \frac{-\mu_0 \mu \bar{h}}{\mu_0}; \quad P = \frac{h_0^3 p}{\mu a^3 h_0};
\]

\[
\sigma = \frac{\sigma}{h_0}; \quad \epsilon = \frac{\epsilon}{h_0}; \quad \mu = \frac{\mu}{B}; \quad C = C a^2
\]  

(16)

and solving the concerned Reynolds equation with the associated boundary conditions

\[
P(1) = 0; \quad \frac{dP}{dR} = -\frac{\mu *}{2} \quad \text{at} \quad R=0
\]  

(17)

we get the nondimensional pressure distribution as

\[
P = \frac{\mu *}{2} \left( 1 - R \right) + 6 \int_0^R \frac{R \, dR}{G(h)}
\]  

(18)

where

\[
G(h) = h^3 + 3\sigma h + 3\alpha^2 \bar{h} + 3\alpha^3 \epsilon + 3\sigma^2 \alpha + \alpha^3
\]

The dimensionless load carrying capacity is given by

\[
\bar{W} = \frac{Wh_0^2}{2\pi \mu a^4 h_0} = \frac{\mu *}{12} \int_0^{\bar{h}} \frac{1}{G(h)} \, d\bar{h}
\]  

(19)

where the load carrying capacity \( W \) is obtained from the relation

\[
W = 2\pi \int_0^R \rho p(r) \, dr
\]  

(20)

The response time in dimensionless form becomes

\[
\bar{\Delta} T = \frac{\Delta \bar{W} h_0^2}{\pi \mu a^4} = \frac{\mu *}{12} \int_0^{\bar{h}} \frac{1}{G(h)} \, d\bar{h}
\]  

(21)

where

\[
\bar{h}_1 = \frac{h_1}{h_0}; \quad \bar{h}_2 = \frac{h_2}{h_0}.
\]  

(22)

3. Results and discussion

Expression for dimensionless pressure \( p \), load carrying capacity \( \bar{W} \) and response time \( \bar{\Delta} T \) are presented in Eqs. (18), (19) and (21) respectively. It is clearly seen that these performance characteristics depend on several parameters such as \( \mu * \), \( \sigma \), \( \alpha \), \( \epsilon \), \( B \) and \( C \). These parameters, respectively, describe the effect of magnetic fluid lubricant, roughness parameters and curvature parameters.

The Eq. (19) suggests that the load carrying capacity increases by .083 \( \mu * \). Setting the roughness parameters \( \sigma \), \( \alpha \) and \( \epsilon \) to be zero one obtains the performance of a magnetic fluid based squeeze film trapped between curved circular plates lying along the surfaces determined by hyperbolic function and secant function. Further, taking the magnetization parameter as zero this investigation reduces to the study of the squeeze film behavior between curved circular plates.

Figs. 2-6 present the variation of load carrying capacity \( W \) with respect to the magnetization parameter \( \mu * \) for various values of roughness parameters \( \sigma \), \( \alpha \) and \( \epsilon \) and the curvature parameters \( B \) and \( C \) respectively. These figures indicate that the load carrying capacity increases significantly with respect to the magnetization parameter. Further, among the roughness parameters the combined effect of the magnetization parameter and skewness is more pronounced.

Figs. 7-9 describe the effect of the standard deviation associated with roughness on the distribution of load carrying capacity. It can be easily observed from these figures that the effect of the standard deviation is considerably adverse, in the sense that the load carrying capacity decreases considerably. This negative effect of \( \sigma \) is little bit less with respect to the upper plate curvature parameter.
In Figs. 10-12 one can have the effect of variance on the variation of load carrying capacity. These figures tell that \( +ve \) decreases the load carrying capacity while \( -ve \) increases the load carrying capacity. Further, it is suggested that the combined effect of the upper plate curvature parameter and the negative variance is significantly positive.

Figs. 13-14 show the effect of skewness on the distribution of load carrying capacity. As in the case of variance here also, \( \varepsilon \ ( +ve ) \) decreases the load carrying capacity while the load increases with respect to \( \varepsilon \ ( -ve ) \). In addition, there is the symmetric distribution of the load.
carrying capacity with respect to the lower plate curvature parameter.

Interestingly, it is noted that the rate of increase in load carrying capacity with respect to the magnetization parameter is more with respect to lower plate’s curvature parameter as compared to the upper plate’s curvature parameter. Lastly, the response time $\Delta T$ follows almost the trends of load carrying capacity.

4. Conclusion

This article reveals that by properly choosing the curvature parameters of both the plates and the magnetization parameter the performance of the bearing system can be enhanced considerably in the case of negatively skewed roughness, especially, when the negative variance is involved. Therefore, this study makes it mandatory that the roughness must be accounted for while designing the bearing system.

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MAGNETOREOLOGINĖ SKYŠČIO PLĖVELĖ TARP IŠGAUBTUŲ APVALIŲ NELYGIŲ PLOKŠTELIŲ

R e z i u m ė

Straipsnyje analizuojama magnetoreologinio skyščio plėvelė, esanti tarp dviejų apvalių išgaubtų, nelygių paviršių plokštelių, kai viršutinė išlenkta plokštė paviršius, apibūdinamas hiperboline funkcija, suartėja su nejuda išlenkta apatinė plokštė, kurios paviršius apibūdinamas sekančioje funkcijoje.


G. M. Deheri, N. D. Abhangi

MAGNETIC FLUID BASED SQUEEZE FILM BETWEEN CURVED ROUGH CIRCULAR PLATES

Summary

It has been sought to analyze a magnetic fluid based squeeze film behavior between two curved rough circular plates when the curved upper plate lying along the surface determined by hyperbolic function approaches the stationary curved lower plate along the surface governed by secant function.

The lubricant used is a magnetic fluid in the presence of an external magnetic field oblique to the radial axis. The transverse roughness of the bearing surfaces is modeled by a stochastic random variable with nonzero mean, variance and skewness. The associated Reynolds equation is averaged with respect to the random roughness parameter. The concerned nondimensional differential equation is then solved with appropriate boundary conditions in dimensionless form to get the pressure distribution, which in turn, is used to get the expression for the load carrying capacity paving the way for the calculation of response time. The results are presented graphically. The results suggest that the bearing system registers a considerably improved performance as compared to that of the bearing system working with a conventional lubricant. It is observed that the pressure, the load carrying capacity and the response time increase with increasing magnetization parameter.

This investigation reveals that although the bearing suffersowing to transverse surface roughness in general,

There are ample scopes for obtaining better performance in the case of negatively skewed roughness by properly choosing the curvature parameters of both the plates.
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ПЛЕНКА МАГНИТОРЕОЛОГИЧЕСКОЙ ЖИДКОСТИ НАХОДЯЩАЯСЯ МЕЖДУ ДВУМЯ ВЫПУХЛЫМИ КРУГЛЫМИ НЕРОВНАМИ ПЛАСТИНКАМИ

Р е з ю м е

В работе анализируется пленка реологической жидкости, находящаяся между двумя круглыми выпуклыми поверхностями. Поверхность верхней выпуклой пластинке, которая приближается к нижней выпуклой пластинке, характеризуется гиперболической функцией, а поверхность нижней пластинки характеризуется сектантной функцией. Смазывающим материалом при этом используется магнитореологическая жидкость, находящаяся во внешнем магнитном поле, наклоненном по отношению к радиальной оси. Шероховатость рабочей поверхности подшипника в радиальном направлении моделируется случайной статистической переменной без нулевого значения ее постоянной и переменной составляющих. Уравнение Рейнольдса, описывающее указанный процесс, усреднено по отношению к случайному параметру шероховатости. С целью установления распределения давления, безразмерное дифференциальное уравнение решено при определенных конечных условиях, результаты использованы для определения несущей способности и времени переходного процесса системы. Полученные результаты представлены в графическом виде. Результаты исследований показали, что характеристики такого подшипника значительно улучшились по сравнению с системами подшипников, использующих обычные смазочные материалы. Установлено, что давление, несущая способность и продолжительность переходного процесса увеличивается при увеличении параметра намагничивания. Исследования позволили обнаружить специфические дефекты подшипника, возникающие из-за поперечных неровностей. Установлено, что при правильном подборе параметров кривизны обеих пластин можно эффективно улучшить эксплуатационные свойства подшипника при негативной асимметрии распределении переменной составляющей шероховатости.

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