Properties of the Retard System Models Based on the Complex Cross Section Multiconductor Lines

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Introduction

Helical and meander slow-wave structures with the super-wide pass-band (from 0 to 10 and more gigahertz) are needful for delay lines, deflection of the electron beam in traveling-wave cathode-ray tubes and other applications [1]–[5]. There are a lot of papers examining properties of helical, meander and other slow-wave structures [4]–[9]. The influence of the various elements of the structures on properties of the structures is examined using various models and methods [6]–[12].

The interactions of the electromagnetic fields, caused by voltages and currents on the adjacent conductors of helical or meander electrodes, act on the electromagnetic parameters of the helical and meander systems and cause changes of retardation factor and characteristic impedance of the system [1, 2, 4], causing amplitude frequency and phase-frequency distortions of signals. In order to reduce the interaction, the bugel-type and the gutter-type helical and meander retard systems with shielding wall elements between neighbor conductors are used [13, 14]. Properties of the bugel-type and the gutter-type systems are examined in [13, 14]. But still there is not enough information about properties of the retard systems with complex cross sections. For this reason, we propose the models and methods for analysis of the bugel-type and gutter-type systems with complex cross section and consider their properties in comparison with helical and meander systems without shielding walls (base type).

As a result, retard system design with the best properties is selected and the way to facilitate manufacturing process of the system is proposed.

Application of the multiconductor line method

The method, based on the multiconductor lines, was developed and is widely used to model wide-pass-band helical structures with rectangular cross-section and meander lines [1, 2, 4, 9, 15]. The principles of modeling are described in [1, 2, 9].

To calculate input impedance and retardation factor versus frequency, dispersion equations and input impedance expressions for homogeneous multiconductor lines are used [1]. The models of the homogenous helical and meander retard systems, based on multiconductor lines, are shown in Fig. 1.

The dispersion equation of the homogenous helical system is given by:

$$kh = \frac{\theta}{2}, \quad (1)$$

where \(k = \frac{\omega}{c_0}\) is the wave number, \(\omega\) is angular frequency, \(c_0\) is velocity of light in vacuum, \(\theta\) is the phase angle between voltages or currents on the adjacent conductors.

Fig. 1. Models of helical (a) and meander (b) retard systems based on multiconductor lines: 1, 3 – shields, 2 – conductors of multiconductor lines, modeling helical (a) or meander (b) wires
Solving dispersion equations (1, 2) we can find values of the wave number $k$ at various values of the phase angle $\theta$.

Further we can find values of the retardation factor $K_R$ and frequency $f$:

$$K_R = \frac{\theta}{kL},$$
$$f = \frac{kc_0}{2\pi}.$$

where $L$ is the step of the helical or meander wires.

The input impedance of the homogenous helical system is given by:

$$Z_{IN} = 1/Y(\theta).$$

The input impedance of the homogenous meander system is given by:

$$Z_{IN} = 1/\sqrt{Y(\theta)\cdot Y(\theta + \pi)}.$$

The dispersion equation of the meander type system and the input impedance expressions of the multiconductor line contain characteristic admittances.

The characteristic admittance of the multiconductor line is given by [16]:

$$Y(\theta) = Y(0) + \left[Y(\pi) - Y(0)\right]\sin^2(\theta/2).$$

Characteristic admittances $Y(0)$ and $Y(\pi)$ or characteristic impedances $Z(0) = 1/Y(0)$ and $Z(\pi) = 1/Y(\pi)$ can be found using numerical methods. We used the Matlab PDE (Partial Differential Equation) Tools software package based on the finite element method [17] to find characteristic admittances $Y(0)$ and $Y(\pi)$.

The characteristic admittances $Y(0)$ and $Y(\pi)$ correspond to the even ($\theta = 0$) and odd ($\theta = \pi$) electromagnetic waves and are given by $Y(0) = c_0\sqrt{C_0(0)C(\theta)}$ and $Y(\pi) = c_0\sqrt{C_0(\pi)C(\pi)}$, where $C_0$ and $C$ are capacitances of a conductor of the multiconductor line with per length at $\varepsilon_\infty = 1$ and $\varepsilon_\omega \neq 1$ correspond, where $i$ is the section number of the multiconductor line cross section.

Using the Matlab PDE Tools software package, based on the finite element method, distributions of potential and electric fields in the models (Fig. 2) can be found. After that we can find capacitances $C_0$ and $C$ using formula [16]

$$C = \int_A \varepsilon_\omega \varepsilon_0 |E|^2 \, dA,$$

where $A$ is the range of integration.

We developed the appendage to the Matlab PDE Tools package for calculations of the capacitances, admittances and impedances.

Using the finite element method, high precision of calculations can be realized [17]. It is important, because accuracy of dispersion evaluation in the retard systems depends on the precision of the characteristic impedance calculations.
Analysis of the complex cross section multiconductor lines

In this section we will analyze properties of the different types of the complex cross section multiconductor lines and retard systems based on them in order to reveal the system with the best properties.

The retardation factor $K_R$ of the homogenous helical system is given by:

$$K_R = 2h / L = \text{const.} \quad (9)$$

According to (9), the retardation factor is constant and the same for all types of the helical retard systems based on the multiconductor lines shown in Fig. 2.

Calculation results of the input impedance versus frequency for the homogenous bugel-type and gutter-type helical retard systems with the complex cross section in comparison with base type retard system are presented in Fig. 3. We can see from Fig. 3 (a) that input impedance change is minimal when the gutter-type helical retard systems with down or up shielding walls are used.

We can facilitate manufacture of the retard systems using the rigid dielectric (for example, sital) base layer. Air or vacuum is in the rest part of the cross section between conductors (permittivity equals 1). Then the cross section of the retard system becomes inhomogenous. Fig 3 (b) illustrates characteristics of input impedance versus frequency when retard system with inhomogenous cross section is used. According to characteristics, input impedance changes are minimal when helical retard systems with down or up shielding walls are used. Due to higher dielectric permittivity, input impedance values are less comparing to the retard systems with air (homogenous cross section) (Fig 3 (a)).

Fig. 3. Characteristic impedance versus frequency of the helical retard system at $L = 1.5$ mm; $d = 0.8$ mm; $r = 0.8$ mm; $r = 0.2$ mm; $w_1 = 0.6$ mm; $w_2 = 0.6$ mm; (a) $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$; (b) $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 7$: 1 – base type helical retard system; 2 – helical retard system with additional bugel shields; 3 – helical retard system with down shielding walls; 4 – helical retard system with up shielding walls

Fig. 4. Retardation factor versus frequency of the meander retard system at $L = 1.5$ mm; $d = 0.8$ mm; $r = 0.8$ mm; $r = 0.2$ mm; $w_1 = 0.6$ mm; $w_2 = 0.6$ mm; (a) $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$; (b) $\varepsilon_{r1} = \varepsilon_{r2} = 1$; $\varepsilon_{r3} = 7$: 1 – base type helical retard system; 2 – helical retard system with additional bugel shields; 3 – helical retard system with down shielding walls; 4 – helical retard system with up shielding walls

According to (2) and (3), the retardation factor $K_R$ of the homogenous meander system is not constant. Characteristics of the retardation factor versus frequency are presented in Fig. 4. Either cross section is homogenous (Fig. 4 (a)) or inhomogeneous (Fig. 4 (b)), retardation factor changes are less, and properties of the meander system are better when the down or up shielding walls are used. The similar conclusion can be done evaluating the input impedance characteristics shown in Fig. 5.

Fig. 5 Characteristic impedance versus frequency of the meander retard system at $L = 1.5$ mm; $d = 0.8$ mm; $r = 0.8$ mm; $r = 0.2$ mm; $w_1 = 0.6$ mm; $w_2 = 0.6$ mm; $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$ (a); $\varepsilon_{r1} = \varepsilon_{r2} = 1$; $\varepsilon_{r3} = 7$ (b): 1 – base type helical retard system; 2 – helical retard system with additional bugel shields; 3 – helical retard system with down shielding walls; 4 – helical retard system with up shielding walls
Manufacturing of retard systems becomes easier if the up shielding walls are used (Fig. 2 (d)). But then uncontrolled gap between shielding wall and the rigid base dielectric layer (Fig. 6 (a)) may appear during manufacture process. Such gap can have unwelcome influence on the properties of a retard system. We can evaluate influence of the gap on properties of the system using formulas:

\[
\Delta(0) = \frac{Z(0) - Z_t(0)}{Z_t(0)} \cdot 100, \quad (10)
\]

\[
\Delta(\pi) = \frac{Z(\pi) - Z_t(\pi)}{Z_t(\pi)} \cdot 100, \quad (11)
\]

where \(Z_t(0)\) and \(Z_t(\pi)\) are input impedance values when \(t_1 = 0\) (metallization between shielding wall and the rigid dielectric base layer is done).

Calculation results of the input impedance of the multiconductor line without gap are shown in Table 1.

**Table 1.** Input impedance calculation results of the multiconductor line without gap

<table>
<thead>
<tr>
<th>(\varepsilon_1)</th>
<th>(\varepsilon_2)</th>
<th>(\varepsilon_3)</th>
<th>(Z(0), \Omega)</th>
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<tr>
<td>1</td>
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<td>3</td>
<td>42.8958</td>
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<td>7</td>
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<td>7</td>
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<td>11</td>
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<tr>
<td>1</td>
<td>1</td>
<td>11</td>
<td>25.4987</td>
</tr>
</tbody>
</table>

Calculation results of \(\Delta(0)\) and \(\Delta(\pi)\) versus gap value and dielectric permittivity value of the base layer are presented in Fig. 7. It can be clearly seen that, when the gap between the shielding wall and the rigid dielectric base layer is very small, unwelcome influence on the properties of the retard system is increasing faster comparing to the influence at higher gap. Such small gap usually is uncontrolled. As dielectric permittivity of the base layer \((\varepsilon_{13})\) is higher, as greater influence of the small uncontrolled gap on the properties of the retard system exists.

As it was mentioned before, the retardation factor of the helical system is constant and depends only on the length dimensions and period of the multiconductor lines. Then using the TEM wave approximation, we have that a small uncontrolled gap between the shielding walls and the rigid dielectric base layer doesn’t have influence on the properties of the system.

**Fig. 6.** Cross sections of the multiconductor lines with up shielding walls: 1 – conductor of the multiconductor line, modeling helical or meander electrode; 2, 3 – shields; 4 – shielding wall

**Fig. 7** Influence of the gap to the properties of the retard system at \(L = 1.5\) mm; \(d = 0.8\) mm; \(r = 0.8\) mm; \(w_1 = 0.6\) mm; \(w_2 = 0.6\) mm; \(l_1 - \varepsilon_1 = \varepsilon_2 = 1; \varepsilon_3 = 3; 2 - \varepsilon_1 = \varepsilon_2 = 1; \varepsilon_3 = 7; 3 - \varepsilon_1 = \varepsilon_2 = 1; \varepsilon_3 = 11\)

**Fig. 8.** Characteristic impedance versus frequency of the helical retard system with up shielding walls at \(L = 1.5\) mm; \(d = 0.8\) mm; \(r = 0.8\) mm; \(w_1 = 0.6\) mm; \(w_2 = 0.6\) mm; \(\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1\) (a); \(\varepsilon_1 = \varepsilon_2 = 1; \varepsilon_3 = 7\) (b): \(1 - t_1 = 0\) mm; \(2 - t_1 = 0.02\) mm; \(3 - t_1 = 0.04\) mm; \(4 - t_1 = 0.06\) mm; \(5 - t_1 = 0.08\) mm; \(6 - t_1 = 0.1\) mm
According to Fig. 8, the input impedance of the helical retard system decreases with decrease of the gap. Also then the input impedance change with frequency is less, and the helical retard system has wider pass-band.

Retardation factor of the meander retard system changes with frequency (Fig. 9). If the gap between shielding walls and the rigid dielectric base layer is less, retardation factor change with frequency is less (Fig. 9). The input impedance of a meander system changes with frequency less than the input impedance of a helical system, but unwell influence of the gap exists as well (Fig. 10). Anyway, input impedance change versus frequency is not significant. Increase of the gap causes increase of input impedance value of the meander retard system.

So, calculation results of the helical retard system input impedance (Fig. 8), meander retard system retardation factor (Fig. 9) and meander system input impedance (Fig. 10) versus frequency confirm that the gap has unwell influence on the properties of the system.

It is possible to avoid appearance of the uncontrolled small gap using metallization layer on the base dielectric layer (Fig. 6 (b)). For this reason, during design and manufacture of retard systems it is recommended to use metallization between the shielding wall and the rigid dielectric base layer (Fig. 6 (b)).

Conclusions

The gutter-type retard systems have better properties than the base and bugel-type retard systems.

We propose to use rigid dielectric base layer and upper shielding walls in order to facilitate manufacturing process of the gutter-type retard system.

Fig. 9. Retardation factor versus frequency of the meander system with up shielding walls at \( L = 1.5 \) mm; \( d = 0.8 \) mm; \( r = 0.2 \) mm; \( w_1 = 0.6 \) mm; \( w_2 = 0.6 \) mm; \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1 \) (a); \( \varepsilon_1 = \varepsilon_2 = 1; \ varepsilon_3 = 7 \) (b): \( 1 - t_1 = 0 \) mm; \( 2 - t_1 = 0.02 \) mm; \( 3 - t_1 = 0.04 \) mm; \( 4 - t_1 = 0.06 \) mm; \( 5 - t_1 = 0.08 \) mm; \( 6 - t_1 = 0.1 \) mm

The possible gap between shielding walls and the rigid dielectric base layer has unwell influence on the properties of a retard system. As gap is less as better properties of the gutter-type retard system are.

Unwell influence on the properties of the retard system increases with decrease of the gap. If dielectric permittivity of base layer (\( \varepsilon_3 \)) is higher, influence of the small uncontrolled gap on the properties of the retard system is greater.

To avoid appearance of the uncontrolled gap, the metallization layer is recommended under the up-type shielding wall.

References

1. Вайниорис З., Кирвайтис Р., Штарас С. Электродинамические задерживающие и отклоняющие системы. – Вильнос: Мокслас, 1986. – 266 с.


We compare helical and meander retard systems based on the multiconductor lines with the complex cross-sections using multiconductor line method. We compare properties of the systems containing shielding elements between the helical or meander conductors and properties of the simple helical and meander systems.

The gutter-type systems have advantages with respect to other systems. When helical or meander electrodes are made on the rigid dielectric layer, the upper-type gutter shield can be used. Then a small uncontrolled gap between the dielectric substrate and the shielding wall can appear. This gap has an unwelcome influence on characteristics (retardation factor and characteristic impedance) of the retard systems. As dielectric permittivity of the substrate is higher, as greater influence of the small uncontrolled gap on the properties of the retard system exists. We propose to use a metal layer on the substrate below the shielding walls of the gutter-type shield. Ill. 10, tabl. 1, bibl. 17 (in English; summaries in English, Russian and Lithuanian).


Рассматриваются свойства замедляющих систем, основанных на многопроводных линиях сложного поперечного сечения. Свойства систем, в которых применены экранирующие элементы между соседними спиральными или меандровыми проводниками, сравниваются с свойствами простых спиральных или меандровых систем.

В случае, когда спиральные или меандровые проводники наносятся на диэлектрическую подложку, целесообразно применять верхний желобковый экран. При этом между экранирующими стенками и диэлектрической подложкой может остаться неконтролируемый зазор. Показано, что такой зазор может оказывать неблагоприятное влияние на характеристики (коэффициент замедления и волновое сопротивление) системы. С целью исключения влияния зазора предложено нанести металлический слой под экранирующей стенкой путем металлизации. Ил. 10, табл. 1, библ. 17 (на английском языке; рефераты на английском, русском и литовском яз.).