Influence of the Thermal EOS Deformations on the Modulation Characteristics

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Introduction

The actual technical parameters of all of the real electron optical systems (EOS) are dependant on the geometry of their elements, which, being influenced by the thermal fields, are deforming, altering the distribution of the electric fields. This is why the simulation of the electric fields commonly with the influence of the thermal fields on the geometry and spatial orientation of the EOS elements may explain certain disagreements between the experimental results and the results of simulations. Therefore, the adequate method for the calculation of the parameters of the actual EOS has to incorporate the evaluation of the thermal fields.

Finite elements method and the tool, which implements this method (ANSYS) was selected to evaluate the influence of thermal fields. Thus the EOS model was made of cathodes, modulator, accelerating electrode and these elements were fixed in the glass ceramics. [1]. It was determined by modelling, that cathodes tend to deform towards the modulator, the surface of the modulator is also slightly approaching the cathodes, and the distance between the accelerating electrode and the modulator is also decreasing. In the working regime the decrease of the distances are as follows: between cathodes and modulator more than 30 µm; between modulator and accelerating electrode up to 30 µm. The elements of EOS are also moving to the x and y directions and these deformations also have influence on the EOS modulation characteristics. Therefore, after calculating the temperature induced deformations of EOS electrodes, the influence of them to the modulation characteristics was evaluated. Consequently the experimental research [2] and the calculation of modulation characteristics were done.

Distribution of the electric field near the cathode

The emission current $I_K$, passing the cathode, can be calculated by Childs–Langmuir law, if the potential distribution near the cathode is known [3]:

$$I_K = S \cdot \frac{4e_0 \sqrt{2e/m}}{9d^2} (V - V_0)^{3/2};$$

here $S$ – area of the cathode emitting surface, $e_0$ – dielectric permeability of vacuum, $e$ ir $m$ – charge and mass of the electron, $V$ – potential of the plane, being at the distance $d$ from the cathode, $V_0$ – potential of electrons in the cathode surface plane.

Distribution of the electric field near the emitting surface of cathodes was calculated by the finite elements method. It was determined, that the influence of electrodes distributed after accelerating electrode is minor to the cathodes emitting current. Therefore, the model for evaluation of potential distribution consists only of cathodes, modulator and accelerating electrode. The space, close to the cathodes, was approximated by the plates with the regular grid of the finite elements (Fig. 1).
Vacuum is a linear medium for electric field, so if one electrode potential is increased by \( n \) times, electric field, created by this electrode, changes in the same way. Therefore it is enough to calculate the influence functions of each electrode. So, this simplification was taken into account and the values of the influence functions of cathodes \( \varphi_{K,ij} \) and accelerating electrode \( \varphi_{G,ij} \) were written as the data files \([4]\). The value of real field at any point is found while multiplying partial electric field data by real voltage values and summing influence of all electrodes: 

\[
U(z,r) = \sum_{i=1}^n \varphi_i(z,r) U_i \ ; \text{where } \varphi_i \text{ and } U_i - \text{the influence function and the potential of the } i \text{ electrode.}
\]

**Emission current**

In the equation (1) the initial velocities of electrons are underrated. But in real EOS the emitting surfaces of the cathodes were supposed to be heated initially, therefore giving some initial temperature dependant velocity to the electrons. Thus the potential difference \( \Delta U_0 \), corresponding to the initial velocity of the electrons was estimated. For this purpose the temperature maps \( T_{K,ij} \) of the emitting surfaces were extracted from the calculated distribution in EOS temperature data. They were interpolated and the values of the temperature \( T_{K,ij} \) in the nodes of the regular grid over the front surfaces of the cathodes. With all this data, the cathode current \( I_K \) was calculated as the arithmetical sum of the elementary cathode currents \( I_{K,e,ij} \) (Fig. 2) in each of the elementary areas \( S_e \) of the grid:

\[
I_K = \sum_{i=0}^{i_{\text{max}}} \sum_{j=0}^{j_{\text{max}}} I_{K,e,ij} . \tag{2}
\]

The average temperature and potential of each elementary area was calculated as follows:

\[
T_{K,e,ij} = 0.25\left(T_{K,ij} + T_{K,i+1,j} + T_{K,i,j+1} + T_{K,i+1,j+1}\right), \tag{3}
\]

\[
V_{e,ij} = 0.25\left(V_{ij} + V_{i+1,j} + V_{i,j+1} + V_{i+1,j+1}\right); \tag{4}
\]

here \( V_{ij} = U_K \varphi_{K,ij} + U_g \varphi_{G,ij} \) (\( U_K, U_g \) – voltage of cathode and accelerating electrode). When the potential distribution near the surface of the cathode is evaluated through the calculation of the influence functions of the electrodes, the emission current for the elementary area of the cathode can be calculated as follows:

\[
I_{K,e,ij} = S_e \cdot A' \frac{2\varphi^{3/2}_{K,e,ij}}{\sqrt{m}} \sqrt{\frac{2\varphi^{3/2}_{K,e,ij}}{m}} E_{K,e,ij} \frac{\varphi^{3/2}_{K,e,ij}}{2}; \tag{5}
\]

here \( \varphi_{K,e,ij} \) – average value of the fluxion of the function of the influence of the elementary cathode area, \( E_{K,e,ij} \) – average value of the electric field strength near the surface of the cathode, can be calculated by the equation:

\[
\begin{align*}
E_{K,e,ij} &= \frac{V_0 - V_{e,ij} + \Delta U_{0,e,ij}}{d}, \\
\Delta U_{0,e,ij} &= \frac{k_B T_{K,e,ij}}{e},
\end{align*}
\]

here \( k_B \) – Boltzmann constant.

**Fig. 2. The algorithm of calculation of the elementary cathode area current**

If the cathode temperature is not changing, the cathode current is able to reach some saturation value, which can be slightly increased, if an electric field applied to the electrodes. This current is calculated by the Richardson-Dushman equation with the Shottky effect. It can be written for the any elementary area as follows:

\[
I_{K,e,ij} = S_e \cdot A' T_{K,e,ij} \frac{k_B T_{K,e,ij}}{e} \varphi_{K,e,ij}; \tag{7}
\]

here \( A' \) – Richardson constant, theoretically equal for every metal, but in fact is dependant on the cathode material (\( A' = 0.85 \times 10^6 \text{ A/m}^2\text{K}^2 \) for oxide cathodes), \( \varphi_0 \) – work of exit of electrons (\( \varphi_0 = 1.5 \text{ eV} \) for oxide cathodes).

Shottky effect in each area \( S_e \): \( \Delta \varphi_{e,ij} = \frac{e^3 E_{K,e,ij}}{4 \pi \varepsilon_0} \). So, calculated value of the elementary area current \( I_{K,e,ij} \) by the
eq. (5) has not to overcome the saturation current $I_{Ks,e,ij}$. The results of calculation of the cathode emission current are given in Fig. 3: in a) and b) figures the current is not limited by saturation current and calculated by eq. (5); in c) figure in some elementary areas the value of the current $I_{K,e,ij}$ overcomes the value of saturation current $I_{Ks,e,ij}$. In these areas the current is equal to saturation current calculated by eq. (7).

![Fig. 3. Alteration of the cathode emission current dependant on the modulation voltage](image)

During the experimental research the initial voltage $U_{Kp}$ was applied to the cathode (voltage of modulator was 0 V) and the voltage of accelerating electrode $U_e$ was changed until the appropriate colour beam was opened. This voltage of accelerating electrode was fixed. Than the voltage of cathode $U_K$ was lowered until zero and the cathode current $I_K$ was fixed. The modulation voltage $U_{mod}$ was calculated: $U_{mod} = U_{Kp} - U_K$. The experiments were done at two initial cathode voltages $U_{Kp} = 100\,\text{V}$ and $U_{Kp} = 150\,\text{V}$. So, the calculations of emission current were done in the same way. First of all, holding the initial voltage $U_{Kp}$, the voltage of accelerating electrode, at which the cathode is open ($I_K \approx 1\,\mu\text{A}$), was adjusted. The algorithm of adjustment of accelerating electrode voltage is given in Fig. 4. Then, changing the voltage of the cathode, the emission current $I_K$ was evaluated.

![Fig. 4. The algorithm of adjustment of accelerating electrode voltage and calculation of modulation characteristics](image)

In the way described above the calculations for the cold and deformed by the heat EOS were performed. In the first case the electrode influence functions near the surfaces of the cathodes were calculated for the optics, drawn regarding to the technical drawings of the assembly. In the second case the calculated temperature induced deformations $u_{xyz}$ were transferred to the cathodes and electrodes of EOS. After forming the new geometry of EOS the influence functions values were calculated again. The currents of the each cathode and the resulting modulation characteristics were calculated for each case for comparison (Fig. 5a). The calculations were performed with the distance $d = 1\,\mu\text{m}$ between the emitting surface of
the cathode and the regular grid with the 5 µm step of the grid. The experimental and calculated results of deformed EOS were compared too. (Fig. 5 b).

![Diagram](image-url)

**Fig. 5.** Deviations of EOS modulation characteristics: a) comparison of calculation results; b) comparison of calculation and experimental results.

**Conclusions**

From the presented results one can conclude that in the case of the deformed optics the emission current is larger than in the case of full correspondence to the technical drawings. The currents of the R and B cathodes are larger for approximately 12%, and the current of the G cathode – up to the 5 %. This is because the temperature induced deformations of the modulator and the accelerating electrode in the area of G cathode are minimal. The calculation results for the deformed optics are much closer to the experimental ones (only up to 5% deviations). Therefore it can be stated that evaluation of the temperature induced deformations give significant accuracy potential to the results of calculation of the cathodes currents.

**References**