Triangle based Image Magnification

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Introduction

Image magnification is a process of obtaining an image at resolution higher than taken from image sensor. Image magnification synonyms are interpolation, enlargement, zooming, etc. To create higher resolution image, previous image must be complemented with new pixels and their intensity must be calculated.

Commonly, image magnification is accomplished through convolution of the image samples with a single kernel – typically the nearest neighbour [1, 2] bilinear, bicubic [3] or cubic B-spline kernel [4] interpolation and training-based algorithms [5, 6].

Triangle based image magnification method

Intensity surface can be explained as landscape with hills and hollows, rides and valleys. Any 3 dimensional (3D) surfaces, with some approximation, can be created from triangles. Look at the image intensity as 3D surface build from triangles where vertexes are image pixels with intensity as z axis (Fig. 1).

Look at small part of intensity surfaces in Figure 2. It is evident, that, in this situation, sharpness of slope is better when triangles are created like in Figure 2b than in Figure 2a. Accordingly the base of both triangles must be diagonal with less intensity gradient. That partially reduce magnification blur.

Fig. 1. Image intensity surface

To magnify image, new image must be complemented with new pixels, added necessary columns and rows of pixels, and calculated new pixels intensity. In this work is made assumption, that new pixels intensity is situated on suitable triangle planes (Figure 1).

During image magnification contours are blurred because of slope sharpness reduction. This is the main problem for all known magnification methods.

So must be located distinctive points on intensity surface and different magnification algorithm applied. There are two things that must be solved: choose the best arrangement of triangles and calculate new pixels intensity.

To explain triangle based magnification method, get four neighbouring pixels from original image and create two triangles planes throws these pixels as vertexes (black dots in Figure 2). These pixels 3D coordinates is known, because they are taken from original image. White dots are new pixels that were inserted to magnify image. Their intensity must be calculated.

Early was decided, that new pixels will situated on two triangles planes. Triangle plane can be formulated as equation of three coplanar vectors over spatial points $M_0(0,0,Z_{00})$, $M_0(S,0,Z_{10})$, $M_0(0,S,Z_{01})$ where x and y coordinates are related with magnification $S$. 

Fig. 2. Two variants of creating pairs of triangles via four pixels
where $x, y, z$ - spatial pixel coordinates, $S$ - magnification, $Z_{xy}$ - image pixel with coordinates $(x, y)$ intensity.

Pixel intensity that is under triangle plane derived from matrix (1) as equation:

$$z_{xy}=((Z_{10} - Z_{00})\cdot x + (Z_{01} - Z_{00})\cdot y + S \cdot Z_{00})/S.$$  \hspace{1cm} (2)

Intensity equation (2) can be formalized as function of pixel coordinates

$$z_{xy} = F_{nm}(x, y),$$  \hspace{1cm} (3)

where $nm$ is index of vertex, in front of base, to mark triangle plane.

As shown in Fig. 2, there are two different ways to draw pair of triangles and calculate pixels intensity.

When magnification $S$ is integer, $x$ and $y$ can get discrete integer values from 0 to $S$. To calculate pixels, with coordinates $(x, y)$, intensity are used formulas (4, 5). When triangles are situated as shown in Fig. 2a,

$$z_{xy} = \begin{cases} F_{10}(x, y), & y \leq x; \\ F_{01}(x, y), & y > x. \end{cases}$$  \hspace{1cm} (4)

Or when triangles are situated as shown in Figure 2b

$$z_{xy} = \begin{cases} F_{00}(x, y), & (S - y) \leq x; \\ F_{11}(x, y), & (S - y) > x. \end{cases}$$  \hspace{1cm} (5)

To decide what is a better way to situate triangles planes, was used intensity vector direction that direction is from pixel with lowermost intensity to pixel with topmost intensity of four pixels ($M_{00}, M_{10}, M_{01}$ and $M_{11})$. Vector can takes eight discrete directions and was marked with number that explained in Figure 3.

Vector mark is main characteristic that allow solving what variant, of two shown in Fig. 2, is better. For example, when vector value is 2, the best way to situate triangles is shown in Fig. 2a, in other hand, when vector value is 4, the best way to situate triangles is shown in Fig. 2b. In comparison of triangles in both of Fig. 2 pictures, is evident that in Fig. 2b, sharpen of triangle plane ($M_{00}, M_{01}, M_{10})$ will increased and plane ($M_{11}$) became flatten, so magnified image sharpness will slightly increase.

**Testing**

This image magnification method was implemented as C++ function in conjunction with free image processing toolkit CImg. For compressed image processing was used Image Magic package.

Scaling down and magnify factors were selected integer numbers. For example scaling factor 3 means that both width and height of image will be divided or multiplied by 3.

Naturally magnified image has some distortions. To measure distortions was calculated root-mean-square-error (RMSE) between original and magnified images. First of all original image was scaled down with scale factor, then image was magnified with the same scale factor, and after that calculated RMSE between original and magnified images for each pixel on each colour layer R, G and B.

For comparison, magnification was executed with three well known magnification methods: block, linear, bi-cubic interpolation and introduced triangle based magnification that will be call “Triangle”.

Two different types of compressed digital image formats JPG and PNG were used. JPG is lossy, but very popular, image format and PNG is lossless format.

For there first test series were selected 9 JPG and 9 PNG images with different size and significant difference in number of small and large objects. Test series with scaling factor 2, 3, 4, 5, 6, 7, 8 was produced with each image.

The second test was the same but with 5000 images with various size and format types for statistic analysis.

**Test results**

First test series of image magnification RMSE is shown in Fig. 4 for JPG images and Fig. 5 for PNG images.
Because of significant difference of RMSE for different images in figures are shown difference from Fig. 5 PNG image magnification RMSE minimal to maximal RMSE values of tested images for different methods and magnification factors. As seen from figures (Fig. 4. and Fig. 5.) “Triangle” magnification method quality is nearly linear interpolation quality (less RMSE is better).

Fig. 6 PNG and JPG image magnification RMSE

“Triangle” magnification method RMSE of JPG and PNG image formats is shown in Fig 6. PNG image magnification distortion is slightly less, because JPG compression add some distortion in to image.

Fig. 7. Motocross

Fig. 8. Belle

Some thing must be explained about RMSE diagrams. For example picture (Fig. 7) RMSE was biggest for all magnification methods and all scaling factors, and picture (Fig. 8) RMSE was least for the same conditions. So great difference in RMSE (from 15 to 25) was found because for testing were selected images with different amount of small details. Landscape, space or human face (Fig. 8) images with big flat areas have smallest RMSE.

There was calculated ANOVA Statistical Significance (p-level) between triangle and each other three methods, that where used for magnification methods comparison (Table 1), with second test series. In Table 1 is shown average RMSE values. Less RMSE is better and Triangle and Linear methods average RMSE are near the same.

Table 1. RMSE statistics of magnification method

<table>
<thead>
<tr>
<th>Method</th>
<th>Triangle</th>
<th>Linear</th>
<th>Block</th>
<th>Bi-cubic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>31200</td>
<td>31200</td>
<td>31200</td>
<td>31200</td>
</tr>
<tr>
<td>Average</td>
<td>19.865</td>
<td>19.803</td>
<td>24.099</td>
<td>22.898</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>10.146</td>
<td>10.035</td>
<td>11.939</td>
<td>11.492</td>
</tr>
<tr>
<td>P-value</td>
<td>-</td>
<td>0.443</td>
<td>0</td>
<td>1.0E-261</td>
</tr>
</tbody>
</table>

During magnification was measured magnification time for all tested methods and scaling factors. Magnification time was measured in milliseconds. Because of different size and coloured or black and white image, magnification speed was recalculated by formula:

\[
P = \frac{\text{width} \times \text{height} \times \text{colour}}{t_m \, \text{ms}},
\]

where \(t_m\) – magnification time, colour – colour layer number (RGB=3, B/W=1). Number \(P\) in formula (6) is a magnification speed with dimension pixels per millisecond, and shows how many pixels of black and white (b/w) image are processing per one millisecond. For colour image that have had 3 colour layers processing time will increase 3 times.

Table 2. Average magnification speed pix/ms of b/w image

<table>
<thead>
<tr>
<th>Magnification Method</th>
<th>Average time (pix/ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P4, 3 GHz</td>
</tr>
<tr>
<td>JPG</td>
<td>8013</td>
</tr>
<tr>
<td>PNG</td>
<td>879</td>
</tr>
<tr>
<td>Linear interpolation</td>
<td>969</td>
</tr>
<tr>
<td>Bi-Cubic interpolation</td>
<td>6849</td>
</tr>
<tr>
<td>Triangle</td>
<td>7937</td>
</tr>
<tr>
<td></td>
<td>426</td>
</tr>
<tr>
<td></td>
<td>3788</td>
</tr>
</tbody>
</table>

In Table 2 is shown average magnification speed (pixels per millisecond) of b/w image on PC Intel Pentium 4 3 GHz (P4 3 GHz) and Intel Pentium III 750 MHz (P3 750 MHz) for fist test series.

Fig. 9. Magnification speed histograms
Triangle method speed is more than 7 times faster than linear and more than 8 times than bi-cubic interpolation methods, with the nearly same quality and only 1.2 times slower than the simplest and fastest block magnification method. The same ratio of magnification time is on computers with different processor speed.

Magnification speed was calculated during second test series that time-span was a few days. Some dispersion was as result of computer multitasking and to get proper results, was build speed histograms that’s show percents from samples count (Fig. 9).

In Fig. 9 are shown magnification speed histograms of 5000 pictures. Statistically triangle magnification speed is more than 8 times faster than linear or bi-cubic. Block, linear and bi-cubic interpolation have had strong peaks, but “Triangle” method have had four peaks.

That is because “triangle” method spends different time to make decision how to create triangles and spend different time for different magnification scale.

Conclusion

Triangle magnification method distortion is nearly linear interpolation method. When magnification factor grows, RMSE grows a bit slower than in linear or bi-cubic interpolation and significant slower than in block magnification (Fig. 4, 5).

Triangle magnification method is faster form 7 to 8 times with the quality nearly linear interpolation methods (Table 2 and Fig. 9).

Triangle magnification method can be implemented on microprocessor or microcontroller with only integer arithmetic, when magnification factor is integer.

References


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