Hybrid Control-based Model Reference Adaptive Control

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Introduction

The switched controller, which is a class of hybrid controller, has been researched in many control systems, such as an automated vehicles highway system [1], robotic control [2], chemical processes [3], traffic management [4], and adaptive control systems [5,6]. In this paper, we apply the switched controller as the adaptive controller to the well-known model reference adaptive control system (MRACS). The switched controller consists of a finite set of analog system and the switches which connect the member of analog system into the closed-loop system as shown in Fig.1. The set of analog controller C1, C2, ....CN will be connected to or disconnected from the closed-loop path suitably by the switch swi(t) which is controlled by the switching controller so as the error e(t) converges to an enough small value in reasonable sense beneath presence of parameter variation of the plant.

The proposed switched controller is implemented and applied to the speed control of one-link arm robot with variation of moment inertia of arm. The experiment results show that the rotation speed of one-link robot’s arm is completely tracked to the desired output of reference model even the moment inertial of arm is varied.

In the next section, the 1st order plant will be considered and their simulation results will be discussed.

1st order plant

CASE I: Let’s consider a first order plant as described in Eq.1 and the parameter K will be treated as variation parameter.

\[ T \frac{dx(t)}{dt} + x(t) = Ku(t). \]  

(1)

Here the two integral controllers will be used as the analog controller C1 and C2 in the switched controller. Then the controlled signal is calculated as follows.

\[ u_i(t) = \int_0^t g_i s_{wi}(\tau)q(\tau)d\tau; \quad i = 1, 2; \]  

(2)

where, gi is a constant gain of analog controller Ci , q(\tau) is the error signal and swi(t) is two values (1 or 0) function determined by switching controller as following equation.

\[ sw_i(t) = \begin{cases} 1 &; C_i \text{ is being connected}, \\ 0 &; C_i \text{ is being disconnected}. \end{cases} \]  

(3)

Thus swi(t) can be considered as a pulse train function. If \( \delta_i(t) \) and \( \lambda_i(t) \) denotes its pulse width and period respectively. Let \( h_i(t) \) denotes the duty rate of each swi(t) then the \( h_i(t) = \delta_i(t)/\lambda_i(t) \). If frequency of switching is higher than the frequency of control signal u(t) enough so then the control signal can be approximated as follows

\[ u(t) \approx \int_0^t (h_1g_1 + h_2g_2)q(\tau)d\tau \]  

(4)

where, \( h_1 \) and \( h_2 \) are the duty rate of switch sw1(t) and sw2(t) mentioned above respectively. Using this control signal in Eq.4 to the plant in Eq.1 then the closed-loop adaptive control system can be described by below:

\[ T \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} + (h_1g_1 + h_2g_2)Kx(t) = (h_1g_1 + h_2g_2)Kr(t). \]  

(5)

As above closed-loop system, that consists of 1st order plant and the integrator, the reference model should be 2nd order system described as follows
By comparison Eq.6 with Eq.5, we can show that the error signal \( e(t) \) between the output of plant and reference model will be converged to zero asymptotically if the coefficient of both equations are the same or Eq.7 is satisfied.

\[
(h_1g_1 + h_2g_2)k = L_m.
\]  

(7)

To obtain the satisfaction in Eq.7 the switching controller will turn the switch to connect an analog controller to the closed loop path and disconnected the behind one from the loop logically by the following switching law. However the limitation of bandwidth frequency of switch and the analog controller is existed so then the considerable small boundary region of error \( \varepsilon \) will be applied to the switching law as the hysteresis width. \( \delta \) is the sampling time of switching controller.

**Table 1. Switching law of switched controller**

<table>
<thead>
<tr>
<th>e(t)&gt;\varepsilon</th>
<th>e(t)&lt;\varepsilon</th>
<th>e(t)=\varepsilon</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW(t)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SW(t-\delta)</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

To investigate the above adaptive scheme a study case of simulation is performed. The following parameters condition is used for simulation.

- Model: \( T = 0.35 \), \( L_m = 1.5 \).
- Plant (DC motor): \( T = 0.35 \), \( K = 0.5 \).
- Hybrid analog controller: \( C_1 = 10/s \), \( C_2 = -10/s \)

The simulation results, the step response of both model and plant are shown in Fig.3 and Fig.4 respectively. Furthermore, the control signal is shown in Fig.5.

**CASE II**

In this case, the variation of both parameters \( T \) and \( K \) of plant in Eq.(1) will be considered. To obtain the response that coincides with the response of 2nd order model described in Eq.6, the adaptation of the closed inner loop is needed. So then a hybrid adaptive controller is added to the adaptive control system as shown in Fig.5.

As Fig.5, the \( C_3 \) and \( C_4 \) are the proportional analog controller with constant gain \( g_3 \) and \( g_4 \) respectively. LPF is a low pass filter that rejects the high frequency signal occurred from switching. The switching law of the hybrid controller in the inner loop is shown in table.2.

**Table 2. Switching law of switched controller**

<table>
<thead>
<tr>
<th>e(t)&gt;\varepsilon</th>
<th>e(t)&lt;\varepsilon</th>
<th>e(t)=\varepsilon</th>
</tr>
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<tr>
<td>SW(t)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SW(t-\delta)</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The simulation is performed by the following conditions:

- Model: \( T_m = 0.35 \), \( L_m = 1.5 \).
- Plant (DC motor): \( T = 2.0 \), \( K = 1 \).
- Hybrid analog controller: \( C_1 = 10/s \), \( C_2 = -10/s \).
- LPF: \( 1/(0.01s+1) \).
The simulation results, the step response of both model and plant are shown in Fig. 6 and Fig. 7 respectively. Furthermore, the control signal is shown in Fig. 8.

**Experiments**

The proposed switched controller is applied to speed control of one-link arm robot and its experiment is performed. The moment inertia of arm will be varied by changing the position of weight attached with the rotating arm. The configuration of experiment is shown as follows.

In the experiment, the step response of rotating arm under two different conditions are investigated. Fig. 3 shows the response when the arm’s inertia is 1.6 [kg.cm²] and Fig. 4 shows the response when the arm’s inertia is 65.5 [kg.cm²]. As both results, the speed of rotating arm (lower) are tracked with the output of model (upper) well.

In the experiment, beneath variation of moment inertia of arm the step response of rotation speed are investigated. The specification of experiment is as follows.

Fig. 10 shows the response when the arm’s inertia is 1.6 [kg.cm²] and Fig. 11 shows the response when the arm’s inertia is 65.5 [kg.cm²]. As both results, the speed of rotating arm (lower) are tracked with the output of model (upper) well.

**Conclusions**

The switched controller applied to MRACS is presented. We show that the parameter adaptation can be done by switched controller. The proposed scheme is applied to the speed control of one-link robot’s arm with
existence of variation of moment inertia. The experiment results show that the rotation of arm is tracked to the desired speed generated by reference model very well.

Table 3. The specification of experiment

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Low pass filter (Reference Model)</td>
<td>( \frac{50}{3s^2 + 24s + 50} )</td>
</tr>
<tr>
<td>DC Motor</td>
<td>( 100 ) 0.11s + 1</td>
</tr>
<tr>
<td>Arm’s inertia</td>
<td>1.2596.8 [kg\cdot cm^2]</td>
</tr>
<tr>
<td>( C_1 ) and ( C_2 )</td>
<td>( \frac{8 \times 10^{-2}}{s} ), ( \frac{-8 \times 10^{-2}}{s} )</td>
</tr>
<tr>
<td>Microcomputer</td>
<td>SH7047</td>
</tr>
<tr>
<td>Power amplifier</td>
<td>100W</td>
</tr>
<tr>
<td>A/D Converter</td>
<td>10 bit</td>
</tr>
<tr>
<td>D/A Converter</td>
<td>10 bit</td>
</tr>
<tr>
<td>Anti-alias Filter</td>
<td>2\textsuperscript{nd} order filter</td>
</tr>
<tr>
<td>Sampling time</td>
<td>1.13 ms</td>
</tr>
</tbody>
</table>

Fig. 10. Condition: Arm’s inertia: 1.6 [Kg\cdot cm^2] (Upper: Output of model, Lower: Rotation Speed)

Fig. 11. Condition: Arm’s inertia: 65.5 [Kg\cdot cm^2] (Upper: Output of model, Lower: Rotation Speed)

References