The Investigation of Eye Tracking Accuracy using Synthetic Images

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Introduction

Human eyes are in constant motion even when a subject tries to fixate a point steadily. Of course, during fixation the movement amplitude is small. Such eye movements are called micromovements. Despite of small amplitude, eye micromovements can help to investigate human cognitive processes and to diagnose diseases of oculomotor system.

All eye movement recording methods involve some compromises, and no method has been devised that is applicable to all situations. The search coil method is one of the few methods that offers the features of high accuracy and large dynamic range. This features are very important for the investigation of eye micromovements. But the method requires putting lens with coil in the eye, which can pose unacceptable risk, especially when it is used in children. Using lens it is a need of anesthesia. Wearing the lens for a long time, it is a possibility to damage a top tissue of the eye, even using a special cure.

In the past few years much research has been directed to non-intrusive gaze tracking using video method[1-2]. Big number of them is devoted for devising eye trackers for human computer interfaces. The application requires eye tracking in free head condition, so optical zoom of eye is used small that eye doesn’t disappear from the image. This cause big position errors and aren’t suitable for micromovements tracking. The manufactures of commercial equipment announce good accuracy of eye recording. But there are no detailed studies about measurement accuracy.

Pupil center detection methods

The detection of pupil center in the image of eye is the most important step for video-based eye tracking method, because full error directly depends on pupil center coordinates errors. The pupil is the largest dark area in the image of eye and it can be distinguished from the surrounding iris by brightness threshold value. If pixel brightness value is less than threshold value, than this pixel is assigned to pupil. If the coordinates of pupil center will be evaluated in integer values of pixels, this cause loss of some information, and method errors will be large. Because we want achieve good accuracy, pupil edge must be located with subpixel accuracy. In the region between iris and pupil brightness from pixel to pixel changes monotonically. It help us to evaluate edge points with subpixel accuracy. In the transit region pixel brightness is fitted to polynomial function versus coordinate. Than pupil edge point coordinates can be evaluated precisely. It is evident that the errors are inverse proportional to brightness gradient.

The pupil center coordinates can be determined from pupil edge points position by some different methods. We used two distinct methods. One of them is widely used in computer vision method, it is points fitting by circle method. Shorter we call it as circle approximation method. Another method is novel. It is based on averaging of coordinates of pupil edge in scan lines. Further we named it as coordinates averaging method.

In the first method to compute the parameters of the circle (center coordinates \( x_0, y_0 \) and radius \( R \)), we apply the method of Chaudhuri and Kundu [3]. This method is based on minimizing the least square error \( J \):

\[
J = \sum_{i=1}^{n} w_i \left[ (x_i - x_0)^2 + (y_i - y_0)^2 - R^2 \right]^2 ; \quad (1)
\]

here the summation is over all edge points \((x_i, y_i)\), with \( i = 1...n \). The least square error can be minimized by setting the derivative of \( J \) with respect to \( x_0 \), \( y_0 \) and \( R \) to zero. Then \( x_0, y_0 \) and \( R \) are given by:

\[
x_0 = \frac{By \cdot Cx - Bx \cdot Cy}{Ax \cdot By - Ay \cdot Bx}, \quad (2)
\]

\[
y_0 = \frac{Ay \cdot Cx - Ax \cdot Cy}{Ay \cdot Bx - Ax \cdot By}, \quad (3)
\]

\[
R = \frac{1}{W} \sum_{i=1}^{n} \left[ (x_i - x_0)^2 + (y_i - y_0)^2 \right]; \quad (4)
\]

where

\[
W = \sum_{i=1}^{n} w_i, x = \frac{\sum_{i=1}^{n} w_i x_i}{W}, y = \frac{\sum_{i=1}^{n} w_i y_i}{W},
\]

\[
Ax = \sum_{i=1}^{n} w_i (x_i - \bar{x}) \cdot x_i, Bx = \sum_{i=1}^{n} w_i (x_i - \bar{x}) \cdot y_i,
\]

\[
Cx = \frac{1}{2} \sum_{i=1}^{n} w_i (x_i - \bar{x})^2 \cdot (x_i^2 + y_i^2),
\]

\[
Ay = \sum_{i=1}^{n} w_i (y_i - \bar{y}) \cdot x_i, By = \sum_{i=1}^{n} w_i (y_i - \bar{y}) \cdot y_i,
\]

\[
Cy = \frac{1}{2} \sum_{i=1}^{n} w_i (y_i - \bar{y})^2 \cdot (x_i^2 + y_i^2).
\]
Illumination artifacts and eyelids may cause deformations of the imaged pupil edge. The influence of these artifacts can be reduced by giving dubious edge points low weight values $w_i$. Initially all the weights are set to unity. After the first step all the edge points are reweighted depending on their deviation from the previously fitted circle, and the calculation of $x_0$, $y_0$ and $R$ is repeated. The weight for each individual point $x_i$, $y_i$ is calculated using this equation:

$$w_i = \begin{cases} 1 - \frac{d_i}{2\sigma}, & \text{for } \frac{1}{2} d_i < 2\sigma, \\ 0, & \text{otherwise;} \end{cases}$$

(6)

where

$$d_i = \sqrt{(x_i - x_0)^2 - (y_i - y_0)^2} - R;$$

(7)

$$\sigma = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - x)^2.$$  

(8)

This procedure of fitting and weighing is executed max eight times or until the changes in results becomes very small.

The complete pupil edge is defined after two steps: (1) horizontal scanning and (2) vertical scanning. Although scanning in both directions allows to get better performance, it is enough to scan in one direction for circle approximation method. In opposite, the second method needs of both scannings.

The image processing stages results are shown in Fig.1. After first stage pupil edge points were detected in scanning lines (Fig. 1a). For lines with pupil the two points were defined. At the second stage the average of edge points coordinates in each scanning line was calculated (Fig. 1b). A set of new points can be approximated by a vertical line. The result of approximation is equation coefficients $v_0$ and $v_1$ of vertical line:

$$y = v_0 + v_1x;$$

(9)

which are obtained by points fitting to line exploiting least squares method.

![Fig 1. Two stages in pupil image processing: a) after pupil edge points detection, b) after pupil vertical axis detection](image)

Second step is analogous to the first, but now scanning in vertical direction is processed. Fitting to line gives us a equation of horizontal line:

$$y = h_0 + h_1x.$$  

(10)

To find the center of the pupil we solve an equation system consisting of equations (9) and (10):

$$\begin{cases} y = v_0 + v_1x, \\ y = h_0 + h_1x; \end{cases}$$

(11)

$$x_0 = \frac{h_0 - v_0}{v_1 - h_1};$$

(12)

$$y_0 = v_0 + v_1x_0.$$  

(13)

For detection of eye ball angular position coordinates one must do calibration. When the changes of coordinates are small the calibration is linear process. To achieve more general results we shall present results without calibration.

Both described pupil center detection methods were implemented in software using C++ programming language.

**Generation of synthetic image sequences**

In order to examine the accuracy of pupil detection methods the synthetic image sequences were generated. They were used as input for images analysis programme. During generation of video files the signal formation in the video sensors was simulated. It was taken into account that the voltage of each pixel is proportional to the average intensity of light striking on the active pixel area (we used value 64 percents of full pixel area). Also the video signal was discretized into 256 voltage levels in order to examine quantisation errors during analog-to-digital conversion.

Every file contained forty frames. We used three different trajectories for imaginary eye movement:

1) the eye had moved straight down for 0.05 of pixel per frame;
2) the eye had moved straight right for 0.05 of pixel per frame;
3) the eye had moved by diagonal line to left-up direction by 0.05 of pixel in horizontal and vertical directions.

The attempt to evaluate the influence of different artifacts in video images was made. Usually odd and even lines of image have a different brightness despite the progressive scan method. So we produced video files with such discrepancy. Sequences of images when pupil is partially closed with eyelid were generated also. For different conditions images with superimposed white noise with various standard deviations ($\sigma$) were created.

In the images without artifacts the full change of brightness between pupil and surrounding iris was 64 units.

**Results**

The produced files with synthetic eye images were processed by video image analysis software with implemented both pupil detection methods. The errors as difference between detected pupil center and actual one were calculated.

At first, we examined influence of pupil brightness threshold on detection errors. We equated the difference between iris brightness and pupil brightness to 100%. The analysis of noiseless images revealed that the threshold value in the range from 20% to 70% has no influence on he standard deviation on error. But the threshold becomes
important when white noise with big standard deviation is superimposed on image. In Fig. 2 the error and its horizontal and vertical components standard deviations were plotted versus threshold value, when white noise $\sigma=5$. We can see from the plot that the minimum error is reached when the threshold value is about 40%. For further analysis we used optimal threshold value.

![Fig. 2. Standard deviations of error versus threshold value (+ – horizontal error; * – vertical error; ◊ – full error)](image2)

Figure 3 shows standard deviation of vertical error relative to white noise standard deviation. The different pupil center detection methods are represented by different lines. The slope of both lines is very close.

![Fig. 3. Standard deviation of vertical error versus standard deviation of noise. – coordinates averaging method; – circle approximation method](image3)

The results of analysis of movement by diagonal line are shown in Fig. 4 and Fig. 5. Here errors of pupil center coordinates are plotted versus actual pupil coordinates.

We have mentioned above that the sequences of images were prepared to analyse the influence of artifacts such as eyelids on eye. Artifacts envoke deformations of detected pupil edge. Figure 6 illustrates how vertical error depends from the visibility of the pupil. When more than 95% of the pupil is visible both methods have approximately the same errors. When closed area of pupil become bigger, coordinates averaging method gives large errors, while circle approximation method still have little errors.

![Fig. 4. Horizontal error versus actual horizontal coordinate of pupil center $x_0$ (+ – circle approximation method; o – coordinates averaging method)](image4)

![Fig. 5. Vertical error versus actual vertical coordinate of pupil center $y_0$ (+ – circle approximation method; o – coordinates averaging method)](image5)

![Fig. 6. Vertical error relative visibility of pupil in percents (+ – circle approximation method, o – coordinates averaging method)](image6)
Discussion

Analysis of results shows that both pupil center detection methods give similar errors. They can be used for pupil center coordinates detection in software for videooculography. Both methods have their advantages and disadvantages. The circle approximation method are more robust to some artifacts. For example, when the eyelid close part of pupil.

Advantages of coordinates averaging method are dominating, when full pupil is open or pupil edge is damaged by small diameter artifacts. This method is faster. Also it is important that the errors are more predictive than in case of circle approximation method.

Actual errors of eye movement measurement system can be evaluated when we now optical zoom of video system. The zoom which allow get calibration factor 2 pixels per eye rotation degree can be easily achieved. From Fig. 3 we can evaluate measurement accuracy, which is better than 1 arc minute in low noise conditions.

Conclusions

The accuracy of videooculography method was investigated. The errors of measurements in low noise conditions can be less than 1 arc minute. The proposed novel pupil center detection method have the same accuracy as approximation by circle method, but has better computational performance.

References


Accuracy of videooculography was tested using synthetic eye images. The novel pupil center coordinates extraction method was proposed. The method exploits averaging of pupil edge coordinates in the same scan line. The novel method accuracy is similar tone of approximation by circle method. The proposed method has better computational effectiveness, but is less robust to influence of artifacts. Errors for both methods increase versus noise standard deviation with the same velocity. However the influence of noise is small, if region between iris and pupil is sharp. The results of study confirmed that method of videooculography is suitable for recording eye micromovements. II. 6, bibl. 3 (in English, summaries in Lithuanian, English, Russian).


Videookulografinio akies judesų matavimo metodo tikslumas buvo ištirtas naudojant sintezuotus akies vaizdus. Pasiūlytas ir ištirtas naujas akies vyzdžio centro nustatymo metodas, taikant vyzdžio krašto taškų koordinačių vidurkinią. Šiuo metodu galima gauti tokia tiksliausia rezultatų kaip ir aproksimavimo apskritimu metodu. Jis spartesnis, tačiau mažiau atsparus artifaktams. Rezultatai rodo, kad abiejų metodų panaikindama didėją, didėjant triukšmo lygiui, tuo pačiu greičiu. Turint statų perėjimą tarp rainelės ir vyzdžio, triukšmo įtaka nėra didelė. Rezultatai parodė, kad videookulografinis metodas gali būti pritaikytas akies mikrojudesiams tirti. II. 6, bibl. 3 (anglų kalba; santraukos lietuvių, anglių ir rusų kalbomis).