Fifth Order Butterworth Low Pass Square-Root Domain Filter Design

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Introduction

Interest in translinear circuits has rapidly increased recently [1, 2]. Translinear circuits have low supply voltage, low power consumption, high dynamic range and low noise performance [2, 3].

Square-root domain (SRD) circuits are subclass of translinear circuits and externally linear internally nonlinear (ELIN) circuits. ELIN circuits are suitable for low voltage applications and low power dissipation [3, 4]. SRD circuits use the MOSFET devices in the strong inversion region of operation, and thus are based on the quadratic relationship between gate-to-source voltage and drain current [5, 6]. SRD circuits use compounding in signal processing. Therefore, SRD circuits have large dynamic ranges at low supply voltages [1, 3]. In the last decades, SRD filtering has become very popular and many studies have been presented in this issue [7–10].

In this study, a fifth order low pass Butterworth filter is designed with two different approaches having same transfer function by using state space synthesis method. First of all, fifth order state space system equations obtained from the transfer function are considered. Secondly, system is considered as compose of two second order and one first order transfer functions. Therefore, three set of system equations are cascaded to obtain a fifth order filter. Both filters are designed by using state space synthesis method and designed circuits are analyzed with PSpice. Obtained frequency, THD and noise analysis results are presented.

Fifth order SRD Butterworth filter state space synthesis

For SRD circuits, state space synthesis method is based on a nonlinear mapping on input and state variables of state space description of a particular transfer function. Input and state variables are equal to simple functions of node voltages. In design procedure, firstly, an appropriate state space description is considered for the filter; next a mapping function is applied to the input and state variables. Mapping function is a kind of square-root function similar to a MOSFET’s current-voltage relation. After circuit equations are obtained, the circuit is designed using square-root blocks, current mirrors, grounded capacitors, and current sources.

Let us consider a fifth order Butterworth low pass filter transfer function as given in (1). In this equation, \( \omega_0 \) is the pole frequency of the filter. According to state space synthesis method [11, 12] initially a state space description of this transfer function is obtained as shown in (2).

\[
H(s) = \frac{Y(s)}{U(s)} = \frac{\omega_0^5}{s^5 + 3.24\omega_0s^4 + 5.24\omega_0^2s^3 + 5.24\omega_0^4s^2 + 5.24\omega_0^5s + \omega_0^5}\,
\]  

\[
\begin{align*}
x_1 &= \omega_0x_2 - \omega_0u_2, \\
x_2 &= \omega_0x_3 - \omega_0u_2, \\
x_3 &= \omega_0x_4 - \omega_0u_2, \\
x_4 &= \omega_0x_5 - \omega_0u_2, \\
x_5 &= -3.24\omega_0x_5 - 5.24\omega_0x_4 - 5.24\omega_0x_3 - 3.24\omega_0x_2 - \omega_0x_1 + \omega_0u + 16.96\omega_0u_2, \\
y &= x_1.
\end{align*}
\]
Nonlinear square-root transformation is shown in (3):
\[
\begin{align*}
\begin{cases}
x_n = v_n = \frac{I_n}{\beta} + V_{th}, \\
u = \frac{I_u}{\beta} + V_{th},
\end{cases}
\end{align*}
\]  
where \( \beta = \frac{\mu_0 C_{ax} W}{2L} \).

Nonlinear mapping shown in (3) is applied to the state space equations (2). After some manipulation nodal equations are obtained as shown in (4).

Fifth order SRD Butterworth filter circuit can be realized using square-root blocks, current mirrors capacitors and current sources. Square-root blocks which are used in proposed fifth order circuits, were proposed in [9].

Proposed fifth order low pass filter, which is a block diagram of (4), is shown in Fig. 1. In this circuit current steering and current mirrors blocks are designed in usual ways. Numbers above current mirror blocks in Fig. 1 imply to the (W/L) ratios of transistors.

\[
\begin{align*}
C_1 v_1 &= \sqrt{I_f I_2} - \sqrt{I_f I_{u2}}, \\
C_2 v_2 &= \sqrt{I_f I_3} - \sqrt{I_f I_{u2}}, \\
C_3 v_3 &= \sqrt{I_f I_4} - \sqrt{I_f I_{u2}}, \\
C_4 v_4 &= \sqrt{I_f I_5} - \sqrt{I_f I_{u2}}, \\
C_5 v_5 &= -3.24 \sqrt{I_f I_5} - 5.24 \sqrt{I_f I_4} - 5.24 \sqrt{I_f I_3} - 3.24 \sqrt{I_f I_2} - \sqrt{I_f I_1} - \sqrt{I_f I_u} + 8.48 \sqrt{I_f I_{u2}},
\end{align*}
\]

where
\[
I_f = \frac{\omega_0^2 C^2}{\beta}, \quad I_{f2} = \frac{4\omega_0^2 C^2}{\beta}.
\]

This circuit equation is realized by using square-root blocks, current mirrors, current sources and capacitors as shown in Fig. 2.

Fifth order SRD Butterworth filter cascade design

In this section, a fifth order SRD filter is obtained by cascade connection of one first order and two second order filters rather than direct implementation as previous section. This concept is formulized in (6)
\[
H(s) = H_1(s) H_{21}(s) H_{22}(s).
\]  

First order SRD filter transfer function and nodal equations are given in (7)–(9).
\[
\begin{align*}
H_1(s) &= \frac{\omega_0}{s^2 + \omega_0^2}, \\
C_1 v_1 &= -\sqrt{I_f I_1} + \sqrt{I_f I_u}, \\
V_{out1} &= v_1.
\end{align*}
\]  

Fig. 1. Fifth order SRD Butterworth filter circuit
Two second order filter transfer functions and circuit equations are:

\[ H_{21}(s) = \frac{\omega_0^2}{s^2 + 0.618\omega_0s + \omega_0^2}, \quad (10) \]

\[ C_{211}v_{211} = \sqrt{I_1I_{212}} - \sqrt{I_1I_{u2}}, \quad (11) \]

\[ C_{212}v_{212} = -\sqrt{I_1I_{211}} - 0.618\sqrt{I_1I_{212}}^+ \]
\[ +\sqrt{I_1I_{1}} + 0.618\sqrt{I_1I_{u2}}, \quad (12) \]

\[ V_{out2} = v_{211}, \quad (13) \]

\[ H_{22}(s) = \frac{\omega_0^2}{s^2 + 1.618\omega_0s + \omega_0^2}, \quad (14) \]

\[ C_{221}v_{221} = \sqrt{I_1I_{222}} - \sqrt{I_1I_{u2}}, \quad (15) \]

\[ C_{222}v_{222} = -\sqrt{I_1I_{221}} - 1.618\sqrt{I_1I_{222}}^+ \]
\[ +\sqrt{I_1I_{1}} + 1.618\sqrt{I_1I_{u2}}, \quad (16) \]

\[ V_{out} = v_{221}. \quad (17) \]

These circuit equations are obtained by using square-root blocks, current mirrors, current sources and capacitors as shown in Fig. 3.

Two second order filter circuits have been same topology given in Fig. 3. In this circuit, X coefficient is equal to 0.618 and 1.618 respectively for (12) and (16).

**Simulation results**

Proposed both direct and cascade fifth order SRD Butterworth filters are simulated in PSpice. Both circuits use supply voltage of 3 V. These filters are set to 517 kHz pole frequency. The simulations are performed using TSMC 0.35 \( \mu \)m CMOS model parameters in PSpice. The values of capacitances of the circuit are selected to be 50 pF. The bias currents of these circuits, \( I_1 \) and \( I_2 \), are set to be 30 \( \mu \)A and 120 \( \mu \)A respectively.

Proposed both fifth order low pass filters gain and phase responses are given in Fig. 4, Fig. 5 respectively.

Then, the output signal’s THD are measured for input voltage amplitude values from 25 mV to 145 mV for both designed filters. Obtained THD results for both circuit topologies are given in Fig. 6.

Noise analysis is also performed. For both cases noise response shows a low pass filter characteristics. For direct design’s noise is 207.016 nV/\( \sqrt{Hz} \) and for cascade connection’s noise is 76.240 nV/\( \sqrt{Hz} \) for inband frequencies.
Conclusions

In this work, starting from one transfer function, two different fifth order low pass Butterworth filters are designed in SRD by using state space synthesis method. Proposed filters use square-root blocks, current mirrors, current sources and capacitances. First one use a fifth order system equations whereas second one use cascade connected one first order and two second order system equations. Proposed filters are simulated by PSpice using 3V supply voltage, 0.35μm CMOS technology parameters. Current sources and capacitances values are selected to 30μA and 50pF respectively. The pole frequency is equal to 517KHz. PSpice simulations are confirmed that both filters are working as expecting. Both filters frequency, THD and noise analysis are performed and presented. According to analysis results, both filters work similarly in terms of frequency responses. However, cascade design has better THD and noise performances due to using less valued current sources.

References