Performance of SC Receiver over Generalized $K$ Fading Channel in the Presence of Imperfect Reference Signal Recovery

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Introduction

In mobile radio communications, due to a multipath propagation, the incoming signal at the receiver is corrupted by the fast fading effect i.e. the random fast fluctuations of the signal envelope [1, 2]. Also, due to the nature of the propagation medium, there can be also random slow fluctuations of the received average signal power (shadowing effect) [1–6]. In certain propagation environments (for example, communication systems with low mobility: an urban area with dense traffic and large number of mobile users which move with small velocity) the simultaneous influence of both fast and slow fading effect appears. In such situations it is necessary to represent a propagation channel by a composite fading model. Several composite models have been presented in the literature ([1] and references therein). Maybe the most known of these models assumes Nakagami-$m$ distribution (i.e. gamma distribution of instantaneous signal-to-noise ratio (SNR)) and lognormal distribution of average signal power. However, a composite probability density function, obtained in this way, is in integral form and it is not convenient for further analysis. For this reason, equivalent gamma distribution, rather than lognormal distribution, is introduced for describing slow fading effect [3–6]. It is mathematically more versatile model and also accurately describes fading shadowing phenomenon. Consequently, obtained composite probability density function follows generalized $K$ ($K_G$) distribution, which proved to be particularly useful in evaluating the performance of composite channels [3–6].

Diversity technique is a communication receiver technique that provides wireless link improvement at relatively low cost by combating the deleterious effect of channel fading and increasing the communication reliability without enlarging either transmitting power or bandwidth of the channel [1, 6–11]. Among the various known diversity combining techniques, selection combining (SC) is perhaps the most frequently used in practice because of its simplicity of realization [1, 6–8], [11]. It is combining technique where the strongest signal is chosen among $L$ branches of diversity system. The criterion for the selection of the branch is the largest value of instantaneous SNR among the branches. That is the reason why all the calculations for receiver performances in this paper will be presented for SC technique at the reception.

In [6] a detailed performance analysis for the most important diversity receivers (SC receiver among them), operating over a composite fading channel, modelled by the $K_G$ distribution, was presented. Expressions for important statistical metrics have been derived. By using them and by considering independent but not necessarily identically distributed fading channel conditions, several performance criteria have been obtained in closed form. Moreover, the average bit-error probability during the detection of binary phase shift keying (BPSK), differential binary phase shift keying (DBPSK) and 16-quadrature amplitude modulation (QAM) signal was studied. However, no phase error during extraction of the reference carrier in phase-locked loop (PLL) circuit was considered in the case of BPSK and QAM signal detection.

Generally, the PLL is used for carrier signal recovery in the receiver. As the receiver is not ideal, a certain phase error appears. The phase error is a difference between the phase of the incoming signal and the phase of the recovered carrier signal in the loop, and this may lead to serious degradation of system performance. It is a statistical process which follows Tikhonov distribution [12–14].

In this paper we discuss the detection of quadrature phase shift keying (QPSK) signals in a composite fading channel, which follows the $K_G$ distribution. The selection combining is applied at the reception, while the branches of the combiner in general are not identically distributed. The imperfect carrier signal recovery from non-modulated pilot signal is taken into account through the phase error that follows the Tikhonov distribution [12–14]. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the
system performance is examined. As a measure of the reception quality the bit-error rate and receiver sensitivity are used. Numerical results are confirmed by Monte Carlo simulations.

The rest of the paper is organized as follows. First, we consider system model and introduce the analytical approach. Then, the numerical evaluation of BER performance and the simulation approach are described. In next section, numerical and simulation results with appropriate discussions are presented. The final section offers some concluding remarks.

**System model**

After propagation through the composite fading channel, signal at the $k$-th branch of SC receiver has the form

$$z_k(t) = r_k(t) \cos(\omega_d t + \Phi_0 + \delta_k(t)) + n_k(t),$$  

where $r_k(t)$ is the envelope of the received signal, $\omega_d$ is the angular frequency of the carrier, $\Phi_0$ is the transmitted phase of the signal, $\delta_k(t)$ is the random phase (the phase noise caused by a fading), and $n_k(t)$ is the additive white Gaussian noise (AWGN) in the $k$-th diversity branch with zero mean value and variance $\sigma^2$. It is assumed that the noise power is the same in every diversity branch and fading is uncorrelated among different branches. Depending on a sent symbol, in the case of QPSK signal transmission, $\Phi_0$ can take following values from the set $\{\pi/4, 3\pi/4, -3\pi/4, -\pi/4\}$. Since the signal is transmitted over a composite fading channel, envelope of the signal in $k$-th input branch, $r_k(t)$, is a statistical process and its instantaneous values fallow generalized $K$ distribution [4]

$$p_k(\rho_k) = \frac{4}{\Gamma(m_m M_k)} \left( \frac{m_m m_s}{\Omega_{sk}} \right)^{m_m + m_s} \rho_k^{m_m + m_s - 1} \times$$

$$\times K_{m_m - m_s} \left( 2 \rho_k \frac{m_m m_s}{\Omega_{sk}} \right), \quad \rho_k > 0,$$

where the second kind modified Bessel function of order $v$ is denoted by $K_v(\cdot)$ [15, Eq. (8.432)] and $\Gamma(.)$ is gamma function [15, Eq. (8.310)]. Parameters $m_m \ (0.5 \leq m_m < \infty)$ and $m_s$ are fading and shadowing shaping parameter, respectively. Larger values of these parameters indicate a smaller fading/shadowing severity. By setting different values of $m_m$ and $m_s$, (2) can describe a great variety of short-term and long-term fading (shadowing) conditions, respectively. For example, as $m_m \to \infty$, $p_k(\rho_k)$ approximates the well known Nakagami-$m$ fading channel model, while for $m_m = 1$ it approaches Rayleigh-Lognormal (R-L) fading/shadowing channel model [1, 6]. Also, for $m_m \to \infty$ and $m_s \to \infty$, (2) approaches the additive white Gaussian noise (AWGN) channel. The average signal power in $k$-th input branch is $r_k^2 = \int_0^\infty p_k(\rho_k) \rho_k \rho_0 \Omega_{sk} = \Omega_{sk}$. The probability density function (PDF) of instantaneous SNR is then

$$p_k(\rho_k) = \frac{2}{\Gamma(m_m M_k)} \left( \frac{m_m m_s}{\rho_0 \Omega_{sk}} \right)^{m_m + m_s} \rho_k^{m_m + m_s - 1} \times$$

$$\times K_{m_m - m_s} \left( 2 \rho_k \frac{m_m m_s}{\rho_0 \Omega_{sk}} \right), \quad \rho_k \geq 0,$$

where $\rho_0$ is average symbol SNR in $k$-th branch. The relation between the average symbol and bit SNR is $\rho_0 = \rho_{0_k} \log_2 M$, where $M$ is the number of modulation levels (in the case of QPSK it is $M=4$), $\log_2(\cdot)$ is the logarithm to base 2 and $\rho_{0_k}$ is average bit SNR.

The chosen branch in SC circuit is the one with the strongest signal. The PDF of the SNR at the output of the combining circuit with $L$ non identical branches can be written as [1]

$$p_{\rho}(\rho) = \sum_{k=1}^L \left( \int_{\rho_{i=1}}^{\rho_{i=k}} p_k(\rho_k) \prod_{i=1}^L F_i(\rho_i) \right).$$

Substituting (3) in (5) and then using relations [16, Eq. (26)] the closed form expression for CDF at the $i$-th branch can be obtained as

$$F_i(\rho_i) = \left[ \frac{\rho_{i=1}}{\rho_{i=k}} p_i(\tau_i) \right]^{L_i} \rho_i dt .$$

In the special case of independent and identically distributed (i.i.d.) branches expression (4) becomes

$$p_{\rho}(\rho) = L \cdot p_{\rho}(\rho) F_{L-1}(\rho).$$

The purpose of the PLL is to estimate the phase of the incoming signal. In ideal case, the estimated phase should be equal to the phase $\delta(t)$ of the incoming signal. However, in practical realizations there is a certain disagreement between the estimated phase $\hat{\delta}(t)$ and the
phase $\delta(t)$ of the received signal. This disagreement is phase error and it is expressed as $\phi(t) = \delta(t) - \hat{\delta}(t)$.

The PDF for this phase error corresponds to Tikhonov distribution [12–14]

$$p_\phi(\phi) = \frac{\rho_{PLL}}{2\pi} \cos \phi, \quad -\pi \leq \phi < \pi,$$

where $I_0(\cdot)$ is modified Bessel function of the first kind and order zero [15, Eq. (8.406)]. Parameter $\rho_{PLL}$ represents the SNR in the PLL circuit and gives the information about the preciseness of phase estimation of incoming signal. It can be assumed $\rho_{PLL}=1/\sigma_\phi^2$, where $\sigma_\phi$ is a standard deviation of the phase error [12–14].

The expression for the conditional bit-error rate (BER) for QPSK signal, as a function of instantaneous symbol SNR, $\rho$, phase error, $\phi$, and phase error, $\phi$, can be presented as

$$P_b(\phi, \rho) = \frac{1}{4} \text{erfc} \left( \frac{\sqrt{2}}{2} (\cos \phi - \sin \phi) \right) +$$

$$+ \frac{1}{4} \text{erfc} \left( \frac{\sqrt{2}}{2} (\cos \phi + \sin \phi) \right),$$

where $\text{erfc}(\cdot)$ is the complementary error function [15, Eq. (8.250)].

The average BER can be obtained by averaging (10) over all possible values of instantaneous symbol SNR, $\rho$, and phase error, $\phi$

$$P_b = \int_{-\pi}^{\pi} \int_{0}^{+\infty} P_b(\phi, \rho) p_\rho(\rho) p_\phi(\phi) d\rho d\phi.$$

**Numerical evaluation**

In order to obtain numerical results, it is necessary to compute values of double integral (11). There is a number of available numerical methods for calculation of these values, but there is no simple equivalent of Gaussian quadrature rules for multiple dimensions [17]. Therefore, to perform numerical cubature as required in this case, we revert to partial Gaussian quadrature rules for $\rho$ and $\phi$ dimensions. This procedure is in general a suboptimal one, but it is intuitive and, in many cases it can prove very efficient. In short terms, the procedure yields a formula

$$\text{BER} \approx \sum_{k=1}^{K} \sum_{n=1}^{N} A_k B_n e^{\phi_n} P_k(\phi_n, \rho_k) p_\rho(\rho_k) p_\phi(\phi_n),$$

where $\rho_k$ and $A_k$ are abscissas and weights, respectively, of well-known Gauss-Laguerre quadrature rules [17]. Abscissas $\phi_n$ and weights $B_n$ are easily obtained by transforming Gauss-Legendre abscissas and weights $P_n$ and $x_n$ [17]

$$B_n = \frac{4}{\pi} P_n, \quad \phi_n = \frac{\pi}{2} (x_n + 1).$$

**Simulations**

Independently of the analytical approach, Monte Carlo simulations were performed, too. The BER values are estimated on the basis of $2 \cdot 10^7$ bit errors. A minimum number of bits, used to evaluate any BER value, is $10^5$. A maximum number of bits, used in simulation, is $2 \cdot 10^9$. Based on the results in the next section, one can notice that there is a very good agreement between numerical and simulation results.

**Numerical results**

Using (8)-(13), one can calculate the average BER for generalized $K$ ($K_{GC}$) fading channel and discuss performances of the receiver for different values of $m_\alpha$ and $m_\upsilon$ parameters, standard deviation of phase noise, $\sigma_\phi$, as well as for different number of diversity branches $L$.

In Fig. 2 the influence of shadowing intensity ($m_\upsilon$ parameter) on BER of QPSK signal detection is presented for different values of fast fading parameter $m_\alpha$. A selection combiner with two branches is used at the reception. Diversity branches are assumed non-identically.
distributed and fast fading parameters differ in each branch (in this case $m_{m1}$ and $m_{m2}$). A phase error standard deviation is $\sigma_{\phi}$. One can notice that, regardless of the values of $m_m$ and $m_i$ parameters, a BER floor appears. Therefore, further increase of $\rho_{0b1}$ has no influence on BER value. This is because some of the received bits can be wrongly detected, due to the error in PLL, even when the power of additive Gaussian noise is approaching zero. For smaller values of $m_m$ (deeper fast fading) BER floor earlier arises, i.e. for larger average bit BER values. It can be noticed that BER decreases with $m_m$ and $m_i$ increasing.

In Fig. 4 one can observe the impact of non-identical fading distribution in diversity branches on system performance. The case of dual branch diversity reception and the different values of fading parameter in the first branch, $m_{m1}$, is presented, while fading parameter of the second branch, $m_{m2}$, deviates by 25% and 50% of $m_{m1}$ value. It can be seen that this effect of non-identically distributed branches achieves the greatest impact on the BER in the case of higher fast fading severity (smaller $m_m$).

The influence of diversity order on the performances of the receiver can be observed from Fig. 5 where dependence of the average BER on $\rho_{0b1}$ is shown for different values of parameter $L$. With the increase of the diversity order, performance of the receiver improves. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity. Power gain is the highest when order of diversity system changes from $L=1$ to $L=2$. For example, in order to obtain the same value of BER $=10^{-4}$, for parameter values $m_{m}=2.5$, $m_{m}=5$. $m_{m}=10$. $m_{m}=0.8m_{m}$, $m_{m}=0.5m_{m}$.
The phase error very strongly impairs system performance. We traced the influence of PLL circuit parameter on the quality of the reception (namely, the phase error standard deviation). The phase error of $\sigma_{\phi}=10^\circ$ already brings the BER floor and the increase of $\rho_{0\beta}$ can not further improve quality of reception. Of course, with the increase of $\sigma_{\phi}$, this BER floor appears for lower $\rho_{0\beta}$ values. It can be concluded that the quality of PLL circuit in the receiver has a crucial importance.

The non-identically distributed fading in branches of SC receiver impairs the system performance. This effect achieves the greatest impact on the BER in the case of higher fast fading severity.

With the increase of the diversity order, performances of the receiver improve. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity.

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References


This paper considers a partially coherent detection of quadrature phase-shift keying (QPSK) signals in a composite generalized $K$ ($K_G$) fading channel. At the reception the selection combining is applied, while the branches of the combiner are not identically distributed. The extraction of the reference carrier from non-modulated pilot signal is performed in a phase-locked loop (PLL) circuit. The difference between received signal phase and extracted reference signal phase is a stochastic variable with Tikhonov probability density function. The influence of the fading parameters, the standard deviation of phase error and the number of diversity branches on the system performance is examined. The bit-error rate is used as a measure of the reception quality. Ill. 6, bibl. 17 (in English; abstracts in English and Lithuanian).


Nagrinėjama dalinė koherentinė signalo, moduliuoto kvadratinė fazinė manipuliacija, detekcija atsižvelgiant į apibendrinantį $K$ ($K_G$) slopinimą kanale. Priemus signalų analizuojamas neantytių signalus. Nenomuliuoto signalo šaltinis atskiriamas fazinėje kilpoje. Stochastinis kintamasis su Tikhonovo tikimybės tankio funkcija yra pagrindinis skirtumas tarp demoduliuoto signalo ir atskirto signalo šaltinio. Ištirta slopinimo parametrų, standartinio nuokrypio fazinės klaidos įtaka. II. 6, bibl. 17 (anglų kalba; santraukos anglų ir lietuvių k.).