Transient Analysis and Modelling of 2nd- and 4th-Order LCLC Filter under Non-Symmetrical Control

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Introduction

The problem how it is possible to obtain sinusoidal voltage at load side under non-harmonic periodical supply from the converters is very important in technical practices. The paper shows possibilities to use either LCLC resonant filter for frequency of fundamental harmonic component, or LC filter tuned for switching frequency. Both filters have to remove higher harmonic components from the supplying voltage to reach the harmonic distortion roughly 5%. Using non-symmetrical control the output voltage of inverter comprises all harmonic components, both odd and even ones. The paper deals mainly with analysis and modelling of 4th order LCLC filter (of the first type) under non-symmetrical supply and with comparing to the other types of filtering. Simulation results as well as experimental verification confirm good quality of output filter quantities, voltage and current.

Basic connection of single-phase inverter with output resonant filter

The single-phase voltage inverter can be realised in principle as full-bridge [1], [2] or half-bridge connection [3] with DC sources, Fig. 1a. For alternative sources there are either single-phase AC-AC converter – type of cyclo-converter (if it is a natural commutation and $f_1>f_2$) or single-phase matrix converter (with a forced commutation and $f_1>f_2$ or $f_1<f_2$), Fig. 1b, [4], [5]. In case of the harmonic sinusoidal voltage of load demand, it is possible to use resonant AC filter tuned to base harmonic, or filter tuned to switching frequency on converter output, Fig. 2a,b.

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Fig. 1. Principle schematic connections of the single-phase half-bridge voltage inverter: a – the single-phase DC-AC inverter supplied from DC sources; b – the single-phase AC-AC inverter supplied from AC sources

Fig. 2. Principle schematic connections of the single-phase voltage inverter and the output filter: a – the output resonant filter with basic resonance frequency; b – the output resonant filter with switching resonance frequency
Transient analysis and modelling of 4th-order LCLC filter under symmetrical control

Output voltage of the inverter contains by wide spectrum of higher harmonic components. Full-width waveform is depicted in Fig. 3a. Harmonic content (odd harmonics, THD = 43.5 %) is shown in Fig. 3b, [6, 7].

Using Fourier theory one can derive relation (1) for basic harmonic amplitude of output voltage of inverter

\[ U_{1M}(\beta) = \frac{4}{\pi} \sin(\beta/2), \]

where \( U_{1M}(\beta) \) is amplitude of fundamental harmonic depending on voltage pulse width \( \beta \); \( U \) is maximum value of inverter input DC voltage; \( \beta \) is voltage pulse width under the range of 0-180°el. deg., whereby

\[ \beta = \pi - \alpha , \] (2)

where \( \alpha \) is control angle oriented from end of half-period to the end of positive voltage pulse.

Considering converter scheme in Fig. 1a and LCLC filter in Fig. 2a with inductor resistance \( r_L \) and capacitor resistance \( r_C \), then the state-space equations can be [8]:

\[
\begin{align*}
\frac{d i_{L1}}{dt} &= \frac{1}{L_1} u(t) - \frac{r_L}{L_1} i_{L1} - \frac{1}{C_1} u_{C1} - \frac{1}{C_2} u_{C2} , \\
\frac{d i_{L2}}{dt} &= \frac{1}{L_2} u_{C2} , \\
\frac{d u_{C1}}{dt} &= \frac{1}{C_1} i_{L1} , \\
\frac{d u_{C2}}{dt} &= \frac{1}{C_2} i_{L2} - \frac{r_C}{C_2} u_{C2} , \\
\frac{d i_{LL}}{dt} &= \frac{1}{L_L} u_{C2} - \frac{R}{L_L} i_{LL} ,
\end{align*}
\] (3)

where \( L_1 = L_2 \rightarrow r_L = r_L; C_1 = C_2 \rightarrow r_C = r_C. \)

After time discretization of system equations using implicit Euler’s methods

\[ \bar{x}_{n+1} = h(A\bar{x}_{n+1} + B\bar{u}_n) + \bar{x}_n, \] (4)

it yields

\[
\begin{bmatrix}
i_{L1}(i+1) \\
i_{L2}(i+1) \\
u_{C1}(i+1) \\
u_{C2}(i+1) \\
i_{LL}(i+1)
\end{bmatrix}
= \begin{bmatrix}
1 & -1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
-1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & -1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
u(t) \\
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} , (5)
\]

where \( A = \frac{1}{L} \begin{bmatrix}
1 & -1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
-1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & -1 & 1 & 0
\end{bmatrix} ; \)

\( i_{L1}, i_{L2} \) are currents through the inductors \( L_1 \) and \( L_2; i_{LL} \) is current through the load \( R; L_L; u_{C1}, u_{C2} \) are voltages of the capacitors \( C_1 \) and \( C_2; J \) is system matrix; \( h \) is step size and \( u(t) \) is output voltage of the inverter.

Transient analysis and modelling of 4th-order LCLC filter under non-symmetrical control

The real output voltage of inverter waveform has a wide spectrum of harmonic components. Using non-symmetrical control the output voltage of inverter (Fig. 4a) comprises all harmonic components, both odd and even ones of Fourier series as it is shown in Fig. 4b, [6, 7].
\[
\frac{U_{IM}(\beta)}{U} = \frac{2\sqrt{2}}{\pi} \sqrt{1 - \cos(\beta/2)} .
\]  
(7)

Considering converter scheme in Fig. 1a and \textit{LCLC} filter in Fig. 2a under non-symmetrical control then the state-space equations are the same as (3), (5) with symmetrical output voltage of inverter \(u(t)\).

**Transient analysis and modelling of 2nd-order LC filter under bipolar PWM control**

The output voltage of AC link inverter [9, 10] is depicted in Fig. 5a. The harmonic spectrum of that is shown in Figs. 5b, 5c.

Fig. 5. The output voltage of the 1-phase inverter under bipolar PWM control (a) and its harmonic content without filtering (b) and its frequency content (c)

The harmonics in the inverter output voltage waveform appear as a sidebands, centred around the switching frequency and its multiples. It follows, that output voltage does not have higher harmonic components around the fundamental frequency. Now is not necessary to use the output resonant filter tuned to fundamental frequency, but there should be used output resonant filter tuned to switching frequency, which is depicted in Fig. 2b.

Considering converter scheme in Fig. 1a and \textit{LC} filter in Fig. 2b then the state-space equations can be written:

\[
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} u(t) - \frac{1}{1} i_L - \frac{1}{R} u_C, \\
\frac{du_C}{dt} &= \frac{1}{C} i_L - \frac{1}{1} u_C - \frac{1}{1} i_L, \\
\frac{di_{LL}}{dt} &= \frac{1}{L} u_C - \frac{1}{L} i_{LL}.
\end{align*}
\]  
(8)

After time discretization of system equations using implicit Euler’s methods

\[
\begin{bmatrix}
i_L(i+1) \\
u_C(i+1) \\
i_{LL}(i+1)
\end{bmatrix} = \begin{bmatrix}
J - hA & -1 & 0 \\
-1 & J - hA & -1 \\
0 & -1 & J - hA
\end{bmatrix} \begin{bmatrix}
i_L(i) \\
u_C(i) \\
i_{LL}(i)
\end{bmatrix} + \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} u(t) \ast h ,
\]  
(9)

where

\[
J = \begin{bmatrix}
-\frac{R}{L} & -\frac{1}{L} & 0 \\
-\frac{1}{C} & -\frac{1}{C} & -\frac{1}{C} \\
0 & -\frac{1}{L} & -\frac{R}{L}
\end{bmatrix} .
\]  
(10)

\(i_L\) is current during the inductor \(L\) of \textit{LC} filter; \(u_C\) is voltage of the capacitor \(C\) of \textit{LC} filter; \(i_{LL}\) is current during the load \(R, L_L\)

**Results of numerical simulation and experimental verifications of transient**

Fig. 6. Simulation circuit half-bridge connection of inverter with \textit{LCLC} filter

Fig. 7. \textit{LCLC} filter output voltage (red) in steady-state with full-wide of impulses (symmetrical, \(\beta = 180^\circ\)el.)
The output capacitor voltage of \textit{LCLC} filter for load disconnect in time at maximum output filter voltage embodies overvoltage for symmetrical control (Figs. 8, 12) and for non-symmetrical control too (Fig. 10). The overvoltage is higher for the (resonant) quality factor $Q$ equal two.

![Fig. 8. Output capacitor voltage of LCLC filter for load disconnect in time at maximum output filter voltage, quality factor $Q$ is equal one (red) and two (black)](image1.png)

![Fig. 9. Output capacitor voltage of LCLC filter under non-symmetrical phase control 165/180°el](image2.png)

![Fig. 10. Output capacitor voltage of LCLC filter for load disconnect in time of maximum output filter voltage under non-symmetrical phase control 165/180°el. for quality factor $Q$ is equal one (red) and two (black)](image3.png)

![Fig. 11. Output capacitor voltage of LCLC filter under symmetrical control with no full-wide of impulses](image4.png)

![Fig. 12. Output capacitor voltage of LCLC filter for load disconnect in time of maximum output filter voltage under symmetrical control with no full-wide of impulses for quality factor $Q$ is equal one (red) and two (black)](image5.png)

As it is shown on Fig. 13 and 14 load current has minor current overshoots in case of load start-up but bring one of the output capacitor voltage.

![Fig. 13. Output capacitor voltage of LCLC filter $u_c$ for two value of quality factor and input current of LCLC filter for no load start up with full-wide of impulses](image6.png)
In case of simulation of 2nd-order LC filter it is evident that output capacitor voltage of LC filter for load disconnect in time of maximum output filter voltage no embodies high overvoltage, Fig. 17. It has only temporary growth of voltage amplitude because saved energy of inductor is only 5% of load energy. This is better choice of filter realization (Fig. 18). The measured voltage of inductive load is shown on Fig. 19.

Conclusions

The \textit{LCLC} resonant filter for frequency of fundamental harmonic component, or \textit{LC} filter tuned for switching frequency has been realised. Both filters have to remove higher harmonic components from the supplying voltage, but output load voltages have had the harmonic distortion roughly 5%. Using non-symmetrical control the output voltage of inverter comprises all harmonic components, both odd and even ones, but output load voltage have had the small harmonic distortion. Simulation results as well as experimental verification confirm good quality of output quantities of the filter, voltage and current.

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References


The paper shows how it is possible to obtain the harmonic sinusoidal voltage on the load side at non-harmonic periodical supplying from the converters. It can be used either LCLC resonant filter for frequency of fundamental harmonic component, or LC filter tuned for switching frequency. Both filters have to remove higher harmonic components from the supplying voltage to reach the harmonic distortion roughly 5 %. The paper deals mainly with analysis and modelling of 4th order LCLC filter (of the first type) under non-symmetrical supply and with comparing to the other types of filtering. Simulation results as well as experimental verification confirm good quality of output quantities of the filter. Ill. 19, bibl. 10 (in English; abstracts in English and Lithuanian).


Aptvelgiama galimybė gauti sinusinį įtampos signalą, kai prijungus apkrovą iš keitiklių gaunami neharmoninės formos įtampos signalai. Tam galėtų būti panaudoti pagrindinės harmonikos rezonansinio dažnio LCLC filtra ir arba persijungimo dažnio LC filtras. Abiem atvejais reikia pažvelgti į aukštesnius harmonikų dedamąsias. Darbė analizuojamos ketvirts eilės LCLC filtras. Modeliavimo rezultatus patvirtina eksperimentinių tyrimų rezultatai. Ill. 19, bibl. 10 (anglų kalba; santraukos anglų ir lietuvių k.).