ECG Research Using Elements of Matrix Analysis and Phase Planes

R. Smidtaite, Z. Navickas
Department of Applied Mathematics, Kaunas University of Technology, Studentu str. 50, LT-51368 Kaunas, Lithuania, phone: +370 67113831, e-mail: rasa.smidtaite@stud.ktu.lt

E. Venskaityte
Department of Kinesiology and Sports Medicine, Kaunas University of Medicine, Sukileliu av. 17, LT-50009, Kaunas, Lithuania, phone +370 61524037, e-mail: evenskaityte@yahoo.com

Introduction

Phase portraits are an invaluable tool in studying dynamical systems [1]. They consist of a plot of typical trajectories in the state space. This reveals information such as whether an attractor, a repellor or limit cycle is present for the chosen parameter value. It is important in classifying the behaviour of systems by specifying when two different phase portraits represent the same qualitative dynamic behavior.

Repeated moderate physical loads lead to adaptation of the organism, not only increasing its resistance to large physical loads but also providing a broad-spectrum protection, for example, against hypobaric hypoxia, toxins, and ionizing radiation [2].

Statistical or analytical methods can be used for analysis of two synchronous time series. Previous studies have shown that statistical methods are not suitable for biosignals [3], therefore, analytical methods should be considered.

Previous studies [4, 5] have shown that discriminant better reveals the relation between two ECG signals than other characteristics. Therefore, the main aim of this paper is to search the ways to purify the information.

In this study the coherence of two ECG parameters (RR and JT intervals) are investigated during physical load. The phase plane represents homogeneity of repolarisation processes in myocardium of increased heart work during physical load.

Theoretical Background

Previous studies made by scientist from all over the world have showed that concatenation has been the object of interest for years. The coherence between two different human body systems or different system parts was explored by many scientists: motor synergy [6], graphical investigation of coherence [7], complexity of biosignals [8, 9]. This study investigates the coherence between two different ECG parameters RR and JT.

Two synchronous time series \( X = (x_0, x_1, x_2, \ldots) \) and \( Y = (y_0, y_1, y_2, \ldots) \), where \( x_n \) and \( y_n \) are real-valued terms, represent ECG measurement data. Mathematical methods based on matrix analysis are applied when the assumption that \( X \) and \( Y \) are determined is made [10]. Using \( X \) and \( Y \) the matrix time series are constructed:

\[
M_n := \begin{pmatrix}
x_n & x_{n+1} - y_{n+1} \\
x_{n+1} & y_{n+1}
\end{pmatrix}, \quad (1)
\]

Each \( n \)th matrix \( M \) from (1) can be expressed as follows [11]:

\[
M^j := \begin{cases}
\lambda_1^j I_1 + \lambda_2^j I_2, & \text{if } \text{dsk} \ M \neq 0, \\
\lambda^j E + (j - 1) \lambda N, & \text{if } \text{dsk} \ M = 0,
\end{cases} \quad (2)
\]

where \( \lambda_1, \lambda_2 \) are eigenvalues of matrix \( M \), \( I_1, I_2 \) are idempotents, \( N \) is nulpotent. Therefore, several characteristics time series are proposed: discriminants (3), discriminant coefficients (4), idempotent coefficients (5):

\[
d_{\text{dsk}} M_n = (x_n - y_n)^2 + 4(x_{n+1} - y_{n+1}) (x_{n+1} - y_{n+1}), \quad (3)
\]

\[
d_{\text{dsk}} \text{Coeff} M_n = \frac{(x_n - y_n)^2 + 4(x_{n+1} - y_{n+1}) (x_{n+1} - y_{n+1})}{\sqrt{(x_n + y_n)^2 + 4(x_{n+1} + y_{n+1}) (x_{n+1} + y_{n+1})}}, \quad (4)
\]

\[
d_{\text{ide}} \text{Coeff} M_n = \frac{(x_n - y_n)^2 + 4(x_{n+1} - y_{n+1}) (x_{n+1} - y_{n+1})}{2\sqrt{(x_n - y_n)^2 + 4(x_{n+1} - y_{n+1}) (x_{n+1} - y_{n+1})}}, \quad (5)
\]

where \( n \) is the number of cardio cycle, \( \alpha \) and \( \beta \) parameters.
Results

The ECG parameters in 12 leads of continuous monitoring during provocative physical load test (Rouffier test) were investigated. The test was repeated four times: 1st day before 1st training session, after 1st training session, after 2nd training session and 2nd day before 1st training session. The recorded ECG was divided into following stages: before physical load the ECG of sportsman’s steady state was recorded (1 min), then the participants of investigation performed Rouffier test (30 squats per 45 s) followed by 2 min of recovery (divided into beginning (1 min) and the end (1 min) of recovery).

Thus the heart needs to overcome different stages of adaptation during physical load. Therefore intrinsic functional adaptation features and their homogeneity can be investigated using cointegration of two time series taken from ECG monitoring during different states of body (rest, physical workout etc.).

The time series of discriminants \(dsk M_n\), discriminant coefficients \(dskCoeff M_n\) and idempotent coefficients \(IdeCoeff M_n\) are calculated from two time series: duration of RR interval taken from the II standard lead and duration of JT interval of the V standard lead.

In order to reduce noise and correct baseline locally weighted linear regression smoothing (LOESS) method is applied (see Fig. 1.) [12]. The smoothing process is considered local because each smoothed value is determined by neighboring data points defined within the span. The process is weighted because a regression weight function is defined for the data points contained within the span. The weight function used for LOESS is:

\[
    w_i = \left(1 - \frac{|k - \hat{k}|}{d(k)} \right)^3, \tag{6}
\]

where \(k\) is predictor value associated with the response value to be smoothed, \(\hat{k}\) are the nearest neighbors of \(k\) as define by the span, and \(d(k)\) is the distance along the abscissa from \(k\) to the most distant predictor value within the span.

To specify the differences in dynamics of \(dsk M_n\) parameter a phase portrait for each sportsman was drawn. Phase portrait reveals how \(dsk M_n\) change velocity depends on \(dsk M_n\). Therefore, y-axis represents derivative of \(dsk M_n\) and x-axis parameter \(dsk M_n\) (Fig. 2).

Fig. 1. Parameter \(dsk M_n\) before and after smoothing

Fig. 2. Phase portrait of coherence dynamic between RR and JT intervals in subject A after the 1st training session (\(dsk M_n\))

Fig. 3. Phase portrait of coherence dynamic between RR and JT intervals in subject A after the 1st training session (\(dskCoeff M_n\))

Fig. 4. Phase portrait of coherence dynamic between RR and JT intervals in subject A after the 1st training session (\(IdeCoeff M_n\))
Two sportsmen with different duration of training experience were compared. Fig. 2, 3 and 4 represent subject A, who has less than 5 years of training experience and Fig. 5, 6 and 7 represent subject B, who has more than 5 years training experience.

The dynamics of RR and JT intervals ratio shown in the following figures represents the interaction of the processes between regulatory and supplying systems depending on the performed physical load.

The obtained results showed that oscillations occur at the onset of investigated electrocardiogram signals. Depending on the stage of the testing the attractors form the coherence dynamics of the investigated parameters, thereafter bifurcation emerge and the dynamics moves and approaches new stable state.

Parameters $dsk_{M_s}$ and $dskCoeff_{M_s}$ reveal very similar information (Fig. 3, Fig. 4) but in different scale. Parameter $dskCoeff_{M_s}$ is more sensitive than $dsk_{M_s}$: discriminant coefficients grow faster than discriminants. Meanwhile, the parameter $IdCoeff_{M_s}$ gives the opposite result comparing with previous ones. While $dsk_{M_s}$ and $dskCoeff_{M_s}$ grow during physical load, the parameter $IdCoeff_{M_s}$ acts on the contrary. The same situation is seen for subject B.

Even though, Fig. 7 differs from Fig. 4 and is not as informative but it does not deny the fact that idempotent coefficient acts opposite to discriminants. Fig. 7 has a different view because a problem related to the situation when discriminant is close to zero (and idempotent coefficient grows very fast) has not been solved yet. This situation is confirmed by linear graph (Fig. 8).

Comparing A and B person we can conclude that amplitude of discriminant and its derivative is much higher in person whom has longer duration of training experience (person B). Residual fatigue features has influence on the decrease of amplitude of these changes observed after training sessions.

### Conclusions

Phase plane of electrocardiogram parameters discriminants revealed that coherence between regulatory and supplying systems in human body on different stages of adaptation to load have dynamic character. The increase of fluctuation and new stable state reflect different metabolic rate depending on the change of the body state.

The limits of such fluctuations for healthy persons as well as for patients are still not known. But the results of...
this study and future works might be useful for diagnostic purposes in patients or for the evaluation of the effect of physical training for athletes etc.

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References


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Electrocardiogram (ECG) is a diagnostic tool that measures activity of the heart. The dynamics of coherence between two different ECG parameters is investigated. Two time series, representing RR and JT parameters, are cointegrated into one time series and then matrix analysis is applied. While studying dynamical systems phase portraits are an invaluable tool. Therefore, to reveal similarities and differences of several characteristics phase portraits are drawn. In this study data of over twenty sportmen are investigated. The view of phase portraits depends on physical load and training. The coherence of two parameters is to be extended to coherence of three parameters in further studies. Ill. 8, bibl. 12 (in English; summaries in English, Russian and Lithuanian).