THE DAMPED FLUCTUATIONS AS A BASE OF MARKET QUOTATIONS

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Abstract

In this article, the author applied the formula of damped fluctuations to explain the process of market quotations. The result shows that assimilation by the market of any new information takes place alongside two simultaneous processes: a sudden wide spread in the quotation values, which then narrows and comes to nothing, and a gradually growing perception by the market of the new price level, that is, the quantitative measure of new information being assimilated.

Keywords: Pricing Model, Market Quotations, Information, Damped Fluctuations.
JEL Classification: G12, G14.

Introduction

This article is a continuation of the author's research into the mechanism of market prices of financial assets (Yandiev, 2010). The study is based on the line of assumptions, in particular: 1) no information comes into the market; 2) existing prices reflect all the known information; 3) brokers receive neither purchase nor sales orders. And the author of the study has concluded the following, including:

1. The market quotation of the financial assets consist of the present value of cash flows expected from the issuer and a short-term credit (Securities Credit, SC) that professional participants extend one other. Then the return on stocks under each individual transaction may be expressed by the following:

   \[ r = R_{PV} + R_{SC} \]  

   \( r \) is financial asset return;  
   \( R_{PV} \) is return on the company’s present value of cash flows;  
   \( R_{SC} \) is return on (present value) a short-term credit, called the Securities Credit, SC.

2. At the speculative trade there is no communication between risk and return.
3. At the speculative trade the prices do not reflect the accessible information.
4. If \( N \) of securities are circulating in the stock market, it means \( N+1 \) of finance assets are actually circulating in there. An additional asset is the securities credit.
5. If the mathematical expectation of return on speculation in financial markets is less than the current rate of return in the Interbank Credit Market, a professional participant will close up his financial activities in financial markets and transfer his capital to the Interbank Market.

Market quotation formation model

Let us analyze through the securities credit prism the model of the origin of a market quotation. For this we will assume that, contrary to our assumptions, on day “zero” there is an infrastructure of financial markets but there is not a single issued finance asset. In other words, no securities are circulated.

Now let us assume that on day 1 there was an issue of ordinary shares of a certain issuer. Shares were placed at the present value of the issuer’s cash flows. Buyers are obviously long-term investors targeting at acquiring the asset at the fundamental value and receiving dividends on a regular basis. This assurance proceeds from the assumption that on the market there is no information and, consequently, there are no grounds for revision of quotations.

So, shares were successfully placed and are on the exchange quotation list. Nevertheless, there won’t be any changes in the share quotation: there is no new information and, consequently, there are no grounds for revision of the present value (the size of cash flows or the discounting rate). In such conditions there may be only investment transactions, which are closed when the investor needs to revise its portfolio. There are no speculative transactions and there cannot be any because there is neither speculative money nor speculators in the market.

Let us go on and assume that on day 2 there come speculative resources on the market (to be more exact, on cash accounts opened in the exchange trading system). A specific feature of such resources distinguishing them from ordinary investments is that from the very start the speculating dealer is ready to...
sustain some loss when conducting speculative operations, hoping for compensation of all losses in the result of the trading session. The amount of money that the speculator is ready to “lose” is crucial because a market quotation is sure to grow (or shrink) only when a bidder deliberately intends to close unprofitable losing transactions. Hence, the bigger is the amount of funds that the dealer is ready to lose during trading, the higher is the market speculative activity.

Let us determine to what extent the dealer may afford to lose some capital to be not sorry for it. For this purpose the author thinks it logical to proceed from the following assertion: “The dealer won’t be sorry to lose money which he can restore from alternative sources”. An alternative source with a minimum risk of the kind is the interbank market. Then, the amount of money that a speculator is ready to “lose” during one trading session is calculated as follows:

$$L = I \times R \times \frac{1}{365}$$  \hspace{1cm} (2)

- $L$ is the “loss”, or the amount of money that the dealer is ready to spend during speculative trading hoping to win, calculated for one day;
- $I$ is the volume of speculative investments (the volume of monetary funds that are held on accounts in an authorized bank at the exchange and are intended for speculations);
- $R$ is the rate of one-day credits in the IM, in shares;

As a result of the arrival on the market of speculative investments speculating dealers will be faced with problems. On the one hand, they must conduct speculative transactions; on the other hand, in the existing conditions it is impossible. In fact, for the initial quotation they will have to take the present asset value (let us remember, it was the placing price). Then, a deviation downwards will imply destruction of the present value, which is against the logic of the speculative game: in each transaction dealers buy/sell at the full price the present value of cash flows, playing within the securities credit limit.

So, one of conditions of speculative trading is that the asset quotation must go up to a level at which dealers will be able to conduct transactions on a bear-tack basis, without destructing the present value. Such quotation growth is possible if dealers deliberately go for unprofitable (losing) transactions until the quotation goes up to the required level. In this way, part of the dealer’s capital transforms into modification (increase or decrease) of the securities credit.

Let us take a short break and note that the dealer obviously tends to save resources and will not target at a constant securities credit increase; he will go as far as is necessary to reach a minimum level sufficient for speculative trading. Therefore, we may allege that in the environment of the set assumptions the size of the securities credit will be always almost equal or slightly above the standard deviation in the asset quotation ($\sigma$):

$$SC \approx \sigma$$  \hspace{1cm} (3)

Then the present asset value may be found as the difference between the average quotation value and the standard deviation (securities credit) in the asset quotation:

$$PV = \bar{S} - SC$$  \hspace{1cm} (4)

Now let us assume that on day 3 there comes on the market a new portion of speculative investments. There will be only one fundamental difference: the “loss” volume will go up. New resources in the market, which may be spent on new transactions, including at unfavorable prices, will increase the number of transactions.

The author found no arguments which could indicate that an increase in the “loss” volume may raise the absolute value of the securities credit.

Hence, we may assert that an inflow (outflow) of speculative resources affects the growth or reduction in the number of speculative transactions closed in the market. If it is assumed that the dealer is constantly targeted at a certain average “loss” value per transaction, then there is a linear and directly proportional connection between the inflow of speculative investments and the number of transactions:

$$L = U \times u$$  \hspace{1cm} (5)

- $U$ is the total securities under which transactions have been closed;
- $u$ is the average “loss” per transaction with one security.

When the Loss ($L$) parameter is disclosed, we come to the formula to calculate the average loss per sold security:
Now let us suppose that on day 4 there appears information on the market (i.e. we ease off our key assumption No. 1). Response of long-term investors and speculators will be different.

For speculators whether or not there is information in the market makes no difference. The influence of informational inflow on the speculative trading (securities credit trading) is secondary to the dealer’s speculative strategy. In other words, during speculative trading professional participants pay almost no attention to new information. Why? Because in terms of basic bids, that is when there is no information altogether, a speculator may freely trade as well as make a success of it. When information comes, the dealer may use it in his speculating (and, basically, the dealer may use information only to adjust his trading strategy) or may choose not to – depending on estimates of the strategy success probability. And in this case, as I have said before, absence or presence of information as such will in no way affect the progress of speculative trading (because in the market there are no private investors, which are a priori less qualified and informed as compared with institutional ones). That is why the widely recognized effective market concept is of no relevance to speculative trading.

As for long-term investors, the influence of informational inflow on the trade of the present value of a company’s cash flows is absolutely evident. Such influence does not last long but is quite intensive. Duration and intensity are related to reflection of new information in the present value. This is a wave-like process as it reflects the market search for a new asset fair value.

Let us take an example. There comes on the market information evidencing that the issuer’s financial position has changed and such changes affect the present value of the issuer’s cash flows. Responding to this, each of long-term investors immediately puts in new quotations for purchase or sale of the issuer’s securities. It is evident that speculative investors cease their speculative trading activities: it is impossible to trade when the size of the first quotation element (that is, the present value of the company’s cash flows) is undetermined and, consequently, it is impossible to track the size of the securities credit. In such conditions speculators may switch on to the present value trading so as to determine its new value as soon as possible, get it fixed at a new level and go back to the securities credit speculation.

But let us go on with the example. Obviously, each professional participant will have its own expert opinion on to what exactly extent (in figures) the size of the present value has been affected by the informational inflow. It means that quotations will be offered within a wide range. Now let us suppose that the transaction has been closed at the price PV+15, i.e. 15 money units higher against the previous price of the present value. From this point the situation may develop in two ways: either the price is regarded by the market as underestimated and then the next quotation will go at a price higher than PV+15, or the price is regarded by the market as overestimated, and then the next quotation will go at a price lower than PV+15.

Suppose that the price is finally regarded by the market as overestimated. It means the market now has the first reference – the ceiling, and the quotation will not go higher than such ceiling. Now the market must determine the bottom of the range within which a new fair price of the present value will be determined.

Suppose that the next transaction has been closed at the price PV+7. From this point the situation may develop in two ways: either the price is regarded by the market as overestimated or as underestimated. Suppose the price has been regarded by the market as underestimated. It means the market now has the second reference – the bottom, below which the quotation will not go.

Once the ceiling and the bottom of the range are determined, the market will attempt to narrow it because the market needs an exact value of the present value and may not accept a range of values. Thus, the market will again start looking for the ceiling (but not above a similar value of the first range) and then, after finding it, the bottom. In this way the market will keep raising and putting the quotation down (each time with a narrower spread) until it finds a value of the present value, most suitable for a majority of professional participants. This process is expressed by mathematically known damped fluctuations, which may be represented graphically as follows:

\[ I \times R \times \frac{1}{365} = U \times u \]

\[ u = I \times R \times \frac{1}{365} \times \frac{1}{U} \] (6)
However, in our case damped fluctuations lead to a change in the quotation, hence, the quotation which exists before information appears on the market, i.e. before fluctuations start, will be different from the quotation, which will be formed when fluctuations are over. Graphically it may be represented as follows (in cases when new information has a positive effect on the quotation value):

![Figure 1. Fluctuation diagram](http://img.encyc.yandex.net/illustrations/bse/fullsize/02826/535660.jpg)

According to many traders in practice the average time of reflection of new information in the asset quotation (the difference between $t_0$ and $t_1$) is 10-15 minutes maximum. Another proof evidencing that reflection of new information takes this period of time is that trade institutors charge a fee for on-line data on the trading process whereas data with a 10-15 minutes delay are provided for free.

Let us resume. As new information appears on the market: 1) the quotation reflects already two types of trading: besides speculative trading it reflects trading on determination of a new level of the present value; 2) in addition to “re-distributing funds managed by another dealer for their benefit” speculators get a new, not less significant motivation to earn on the quotation fluctuations resulting from the process of determination of a new value of the present value.
Period T-0 – T-1: there is a stock infrastructure but there are no securities in circulation.

Period T-1 – T-2: at the time point T-1 stocks of a certain company are initially placed at a price of 100 units; then, until T-2 transactions may be closed but the quotation will remain unchanged.

Period T-2 – T-3: at the time point T-2 speculative investments come on the market, speculators form a securities credit and start speculative trading; the trading process is represented by local random changes in the quotation.

Period T-3 – T-4: at the time point T-3 new information No. 1, related to the issuer, appears on the market; the market participants immediately start looking for a new asset price level; the trading process is represented by damped fluctuations with a transfer onto a new price level, in this case, to a higher one.

Period T-4 – T-5: by the time point T-4 new information No. 1 is already fully reflected in the quotation and speculators again go back to speculative transactions.

Period T-5 – T-6: at the time point T-5 new information No. 2 appears on the market; as a result the existing quotation will have to be revised (in this case the quotation value has been reduced).

Period T-6 – T-7: by the time point T-6 new information No. 2 is already reflected in the quotation and speculators again go back to speculative transactions.

Period T-7 – T-8: new information No. 3 appears on the market; the market starts looking for a new price level, which is represented, as I have noted before, by damped fluctuations.

Period T-8 – T-9: during this period speculative investors continue speculative trading.

Now let us suppose that on day 5 private investors (clients of brokerage companies) come onto the market. By introducing this element we make our model close, to the extent possible, to practical realities.

Due to a multitude of factors (due dates, there are available resources, intention to diversify capital, sudden desire to make money, completed forecast, an emotional outburst, etc.) clients put in purchase/sale bids, which have nothing to do with the real situation regarding assets of the issuer’s company. Besides, a non-institutional investor would need much more time to assimilate new information than is needed for a professional participant; in other words, new information that has already been assimilated by professional participants will be analyzed and evaluated by non-institutional investors for another day or two, or even a week. As a result the market quotation fluctuations no longer provide for visual reflection of whatsoever that could be logically explained and the impression one can get (known to all who have ever watched quotations) is that of compete chaos.

The only thing that can be reflected in such conditions in a quotation is chaotic tossing of brokers’ clients. Along with that, the earlier analyzed basic behavioral motivations of both speculators and long-term investors become weaker. It is of key importance to be able to guess what the majority will do and make money on it. In other words, by simulating the securities credit concept in conditions of practical realities, we have arrived at the generally known assertion, which was made at their time by J. Keynes, P. Samuelson and other famous scientists and experts only on the basis of experience and personal market observations – find the trend and jump into it.

Model of the market quotation pricing during assimilation of new information

Let us go back to the damped fluctuations structure. It is known that the formulas used for description of damped fluctuations have been adapted by analogy to electric circuits and are now effectively
used in electricity. We may respectively assume that damped fluctuation formulas may as well be adapted to financial market conditions to be used in practice, for example, in retail robot programming.

Let us take the basic formula of damped fluctuations:

\[ x = A_0 e^{-\beta t} \sin(\omega t + \phi_0) \]  

- \( x \) is the quotation value at damped fluctuations;
- \( A_0 \) is the initial maximum amplitude of quotation fluctuations;
- \( e \) is a fixed number, taken for simple calculations as equal to 2.71;
- \( \beta \) is the damp coefficient;
- \( t \) is the time after the start of fluctuations;
- \( \omega \) is the normal cyclic frequency of the system fluctuations;
- \( \phi_0 \) is the initial fluctuations phase (for the sake of simplicity we take it equal to zero).

The most topical values (in the sense that it is difficult to see at once what can be taken to replace them) of the above parameters of formula 7 are the cyclic fluctuation frequency (\( \omega \)) and the damp coefficient (\( \beta \)). Let us analyze their elements.

\[ \omega = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}} \]  

and

\[ \beta = \frac{r}{2m} \]  

Let us specify the meanings of \( k, r, m \) and \( A_0 \):

- \( k \) is the elasticity coefficient;
- \( r \) is the friction coefficient;
- \( m \) is the object weight.

Now we must look for adequate stock market parameters to replace the above ones.

**a) Elasticity coefficient.** This coefficient is defined as the ratio of the tension force to the leg stretching. In other words, the coefficient shows how much force it takes to stretch the object by one length unit. Speaking about the stock market, this parameter may be replaced in my opinion with the ratio of the average “loss” under the transaction (see formula 6) to the quotation change:

<table>
<thead>
<tr>
<th>Original parameter</th>
<th>Adapted parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension force (required to stretch the object)</td>
<td>Average “loss” under the transaction (i.e. how much we put to “stretch” the quotation)</td>
</tr>
<tr>
<td>Object stretching</td>
<td>Quotation change</td>
</tr>
</tbody>
</table>

Then:

\[ k = \frac{u}{S_1 - S_0} \]

**b) Friction coefficient.** In the simplest case, this coefficient is the ratio of force that needs to be taken to make the object move to the normal weight of the object. It must be noted that here we speak in fact about two contacting physical objects, which due to their contact are difficult to move with respect to one another.

The closest in essence in my opinion is the combination of two parameters: the asset return and the stock index return. Of course, they are not in a physical contact but we can see they are statistically connected: there is a correlation coefficient that shows the degree of the asset return change relating to the index return as if there were in a physical contact.

So the friction coefficient may be replaced with the coefficient of correlation between the asset return and the stock index return.

\[ \text{Source: Full Course of Lectures in Physics, http://physics-lectures.ru/} \]
\[ k = K_{korr} \]  \hspace{1cm} (11)

- \( K_{korr} \) is the coefficient of correlation between the asset return and the index return.

\( c) \) Object weight. The closest in essence in my opinion is the Present Value of Forecast Cash Flows of the Issuer.

\[ m = PV \]  \hspace{1cm} (12)

- \( PV \) – is the present value of the issuer’s forecast cash flows.

\( d) \) Initial maximum amplitude of fluctuations. Let us add that with regard to financial markets \( A_0 \) may be replaced with the quotation value at the time new information appears on the market.

\[ A_0 = S_0 \]  \hspace{1cm} (13)

Now that all parameters are found, we must elaborate equation 7 so that it leads to assimilation of new information, that is, to changes in the initial quotation level and not just to damping of fluctuations and return to the previous value. For this equation \( 7 \) must be supplemented with an element reflecting the gradually growing perception of the market of the new price level. It is clear that the higher is the \( t \) parameter, the higher is the value of correction resulting from this formula element. We may suppose that such increase will be directly proportional and then:

\[ x' = x + at \frac{1}{c} \]  \hspace{1cm} (14)

- \( x' \) is the new quotation value (after new information is assimilated);
- \( a \) is the increase in the original quotation, reflecting the quantitative measure of new information;
- \( c \) is the time during which new information will be fully assimilated by the market.

Formula 14 may be as well presented in detail:

\[ x = A_0 e^{-\beta t} \sin(\omega t + \phi_0) + at \frac{1}{c} \]  \hspace{1cm} (15)

Formula 15 shows that assimilation by the market of any new information takes place alongside two simultaneous processes: a sudden wide spread in the quotation values, which then narrows and comes to nothing, and a gradually growing perception by the market of the new price level, that is, the quantitative measure of new information being assimilated.

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